Part VI

Appendices

Appendix A
Computer Programs for MDS

Several computer programs for doing MDS exist, some of which are included in major software packages, and others are in the public domain. The list of programs we discuss in this appendix is not exhaustive, although we have tried to find the most important ones in terms of options and availability at the time of writing. Each program is described briefly. We also give an example of how a simple MDS job is run in each program. Where possible, we provide a subset of the commands and keywords needed for running a simple MDS analysis.

MDS is very much a visualization technique. Fortunately, the graphical capabilities of modern PCs have improved drastically over the years. Therefore, we place more emphasis on the graphical representations provided by MDS programs. In addition, we found two programs (GGVIS and PERMAP) freely available on the Internet that show interactively how the MDS solution is obtained. To reflect the development, we have organized the remainder of this appendix into three sections: the first section discusses two interactive MDS programs, the second section is focused mainly on commercial statistical packages that have high-resolution graphics, and the third section treats MDS programs that do not have high resolution graphics and have mostly been developed in the early days of MDS.

Table A.1 gives an overview of the properties of each of the programs. The MDS models in Table A.1 denote: (a) ordinal MDS with the primary approach to ties; (b) ordinal MDS with the secondary approach to ties; (c) ordinal MDS, using rank-image transformations; (d) interval MDS, $a + b \cdot p_{ij} = d_{ij}(\mathbf{X})$; (e) ratio MDS, $b \cdot p_{ij} = d_{ij}(\mathbf{X})$; (f) splines; (g) polynomial regression, $a+b\cdot p_{ij}(\mathbf{X})+c\cdot p_{ij}^2(\mathbf{X})+\cdots = d_{ij}(\mathbf{X})$; (h) power, $p_{ij} = a\cdot d_{ij}^b(\mathbf{X})$,

which is equivalent to a linear MDS on logged proximities; (i) mixed models, for example, ordinal (unconditional) MDS for matrix 1 and linear MDS for a copy of matrix 1 stored in matrix 2. Note that (h) is but a linear MDS on logged proximities.

One general warning for the use of all programs is in place: many programs have rather weak convergence criteria, which may cause the program stopping the iterations too early and give suboptimal solutions. To be on the safe side, we advise to stop the iterative process only if the difference in two subsequent loss function values (usually Stress, or S-Stress) is smaller than 10^{-6} and setting the maximum number of iterations to 100 or more.

To illustrate the setup of a program, we use the artificial data on the ranking of pairs of politicians in Tables 9.4 and 9.3.

A.1 Interactive MDS Programs

With improving speed and graphical capabilities of modern computers, it becomes possible to animate the way in which MDS solutions are obtained. In this section, we discuss two of these programs, GGVIS and PERMAP. We call these programs an interactive form of MDS because they allow us to manipulate the MDS options by an easy user interface. Any change of MDS options usually leads to animations showing the changes leading to an optimal configuration. In such a way, you can test the stability of the solution interactively, for example, by eliminating points, changing the MDS model, rearranging points to check for local minima, and so on. These programs stay close to the exploratory nature of MDS with an emphasis on visualization.

GGVIS

GGVIS is a standard plug-in for MDS that comes with the GGOBI visualization software. It is freely available from the Internet and can be run as a standalone application or within the statistical programming environment R (also freely available). GGOBI visualizes rectangular two-way-two-mode data allowing an interactive grand tour through high-dimensional spaces, labeling, glyphing, connecting edges, and the like. GGVIS uses many of these options but is tailored for MDS. For an extensive discussion of GGVIS, we refer to Buja and Swayne (2002).

A nice feature of GGVIS is that a change of options has immediate effects on the solution. Thus, the user can see in real-time, for example, how the configuration changes from a metric to a nonmetric solution. Although the emphasis of GGVIS is on metric MDS, it also allows for ordinal transformations. Two options in GGVIS are unique. First, you can set interactively a power transformation of the dissimilarities and the resulting distribution is

TABLE A.1. A summary of several MDS programs; + stands for Yes or indicates that option is available, – shows that option is not available, n.a. means not applicable, and mem indicates that the maximum number of objects depends on memory available.

	Ggvis	Permap	Alscal	Newmdsx	Proxscal	SAS	Statistica	Systat	Fssa	Kyst	Minissa	Multiscale
Platforms												
Version	1.0.0b	11.3	n.a.	4.0.4	1.0	n.a.	4.5	11	3	2a	1	n.a.
Standalone program	+	+	+	+	–	–	–	–	+	+	+	+
In larger package	+	–	+	–	+	+	+	+	–	–	+	–
Commercial	–	–	+	+	+	+	+	+	–	–	+	–
MS-Windows	+	+	+	+	+	+	+	+	–	+	–	+
Macintosh	–	–	+	–	+	+	+	+	–	+	–	+
Graphical user interface	+	+	+	+	+	+	+	+	–	–	–	–
High resolution graphics	+	+	+	+	+	+	+	+	–	–	–	–
Dynamic graphics	+	+	–	–	–	–	–	–	–	–	–	–
General features												
Minimizes Stress	+	+	–	+	+	+	+	+	–	+	+	+
Minimizes S-Stress	–	+	+	–	–	+	–	+	–	–	–	–
Minimizes alienation	–	–	–	+	–	–	–	+	+	–	+	–
Maximizes likelihood	–	+	–	–	–	–	–	–	–	–	–	+
Max. number of objects	mem	200	100	100	mem	mem	90	mem	50	60	100	mem
Min. number of objects	2	2	4	2	2	2	2	2	3	3	2	3
Max. dimensionality	n-1	4	6	10	n-1	n-1	9	5	10	6	10	n-1
Processes rectangular data	+	+	+	+	+	–	–	+	–	+	+	+
Allows for missing data	+	+	+	+	+	+	+	+	+	+	+	+
Offers Minkowski distances	+	+	–	+	–	–	–	+	–	+	+	–
Allows for weights w_{ij}	+	+	–	–	+	+	–	–	–	+	–	–
MDS models												
Ordinal, prim. appr. ties	–	+	+	+	+	+	+	+	–	+	+	–
Ordinal, sec. appr. ties	+	–	+	+	+	+	–	–	+	+	+	+
Ordinal, rank-image	–	–	–	+	–	–	–	+	+	–	+	–
Interval	–	+	+	–	+	+	–	+	–	+	–	+
Ratio	+	+	+	–	+	+	–	–	–	+	–	+
Absolute	+	–	+	–	–	+	–	–	–	–	–	+
Splines	–	–	–	–	+	–	–	–	–	–	–	+
Polynomial regression	–	–	+	–	+	–	–	–	–	+	–	+
Power	–	–	–	–	+	–	–	+	–	–	–	–
Mixed models	–	–	–	–	–	–	–	–	–	+	–	–
Special models												
Split, by row	–	–	+	+	–	+	+	+	+	+	+	+
Split, by row and by col.	–	–	–	+	–	–	–	–	–	+	+	–
Split, by matrix	–	–	+	–	+	+	–	+	–	+	–	–
Asymmetry models	–	–	+	–	–	–	–	–	–	–	–	–
Weighted Euclidean model	–	–	+	+	+	+	–	+	–	–	+	+
Generalized Euclidean model	–	–	+	+	+	–	–	–	–	–	+	–
External variables	–	–	–	–	+	–	–	–	+	–	–	–
Constrained solutions	–	+	+	+	+	+	–	–	–	+	+	–

FIGURE A.1. Three different situations of the MDS settings windows of GGVIS with four tabs at the top.

shown in a histogram. A power may be chosen such that the transformed dissimilarities have a uniform distribution (for an application, see Section 9.7). The second unique option is to set the weights that are applied to the errors to a power of the dissimilarities. Choosing a large positive power emphasizes the correct display of large dissimilarities, whereas a large negative power mostly ignores large dissimilarities and emphasizes the proper representation of small dissimilarities (see Section 11.3).

We tested a beta version of GGVIS. To use GGVIS, you first have to set up a data file that GGVIS can process. Below, we present a sample file in XML. Once the data are read, move to the MDS module by choosing Tools > GGVIS (MDS).... This brings up the window shown in Figure A.1.

Here, you can move to the fourth tab (or directly click on the "Run" button). It opens a window (see middle panel of Figure A.1) where you can change the default parameter settings for the dimensionality of the MDS space and the stepsize for the iterations. For our politicians data, we would set the dimensionality to 2, changing it from the default value of 3. To get an ordinal rather than a metric MDS solution, we would then press the pull-down menu "Metric MDS" in the middle of the window, where we can click on "Nonmetric MDS". Then, click on the "Run MDS" button which starts the program showing how Stress is minimized and how the data are weighted (see windows in the middle of the right panel of Figure A.2). The ordinal MDS solution is given in Figure A.2.

The "Run MDS" button is a toggle. You may, for example, experiment with different stepsizes, metric vs. nonmetric MDS, different weights and so on, and rerun the MDS. From the "Reset" menu, you can reinitialize or

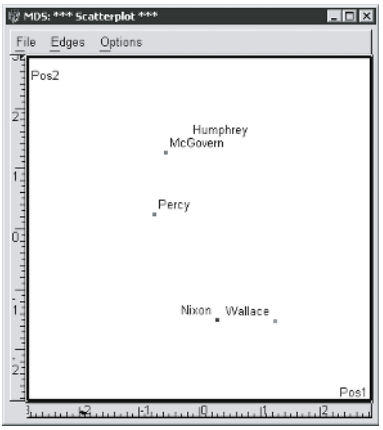

FIGURE A.2. Solution obtained by GGVIS.

scramble the MDS configuration, and from the "View" menu you can view the Shepard diagram.

GGVIS reads its data in XML format. Below you find an example. XML commands are written between <>-signs. Every command that is opened, for example, `<description>`, should also be closed again `</description>`. The first two lines define that these data belong to GGOBI.

```
 <?xml version="1.0"?> <!DOCTYPE
ggobidata SYSTEM "ggobi.dtd">

<ggobidata count="2">
<data name="Politicians">
<description>
Example data set to illustrate {\sc Ggvis}
</description>
<variables count="0">
</variables>
<records count="5" glyph="fr 1" color="3">
<record id="1"  label="Humphrey"  color="1">  </record>
<record id="2"  label="McGovern"  color="3">  </record>
<record id="3"  label="Percy"     color="3">  </record>
<record id="4"  label="Wallace"   color="2">  </record>
<record id="5"  label="Nixon"     color="0">  </record>
</records>
</data>

<data name="dissimilarity">
<description>
 Dissimilarities (rank orders)
</description>
<variables count="1">
  <realvariable name="Dissimilarity" nickname="D" />
</variables>
<records count="10" glyph="fr 1" color="0">
```

```
<record source="1" destination="2">  1 </record>
<record source="1" destination="3">  5 </record>
<record source="1" destination="4">  7 </record>
<record source="1" destination="5">  6 </record>
<record source="2" destination="3">  2 </record>
<record source="2" destination="4"> 10 </record>
<record source="2" destination="5">  8 </record>
<record source="3" destination="4">  9 </record>
<record source="3" destination="5">  4 </record>
<record source="4" destination="5">  3 </record>
</records>
</data>

</ggobidata>
```

GGVIS uses the following commands:

- `<ggobidata count="2">` says that what follows are two data sets specific for GGOBI.
- `<data name="Politicians">` specifies that the data defined here are called 'Politicians'.
- `<description>` allows a description of the data.
- `<variables count="0">` indicate that the rows defined below have no variables. If, for example, `count=1` then one lines should follow defining the variable name and nickname by `<realvariable name="Variable 1" nickname="V1" />`. For more variables, add more lines.
- `<records count="5" glyph="fr 1" color="3">` specifies that five records follow with a certain form of glyph type and color.
- `<record id="1" label="Humphrey" color="1"> </record>` defines the first record to have label 'Humphrey' and a specific color. If there is at least one variable, then their values should be specified before `</record>`.
- `<record source="1" destination="2"> 1 </record>` defines a single dissimilarity for objects 1 and 2. For all available dissimilarities, a single record should be specified indicating the row number and their column number. Missing dissimilarities are obtained by omitting the records for the missing pairs of objects.

More information can be found on the GGOBI website http://www.ggobi.org; E-mail: ggobi-help@ggobi.org

PERMAP

PERMAP is one of the few interactive MDS packages available. It allows users to interact directly with an MDS solution, move objects in the solution space, remove certain objects, and change MDS options. PERMAP is not built on any of the previously existing MDS software and can be freely downloaded from the Internet.

The program has a wide range of options, some of which are unique to PERMAP. It allows for ratio, interval, and ordinal MDS, the latter using the primary approach to ties. It can minimize several MDS loss functions,

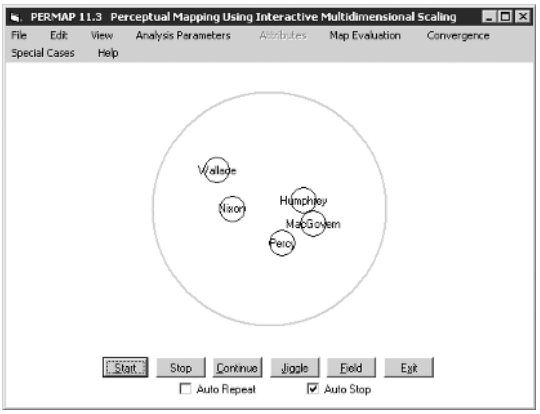

FIGURE A.3. Screen shot of PERMAP.

including Stress, Stress-1, S-Stress, and MULTISCALE. In addition, weights can be specified for every dissimilarity. PERMAP uses the general Minkowski distances that include Euclidean, city-block, and dominance distances. The program can also compute dissimilarities from a rectangular data matrix. A summary of the MDS solution can be saved into a text file. PERMAP comes with an extensive documentation aimed at the nonexpert.

A user-friendly option of PERMAP is to drag objects away from the solution into a "parking lot" to exclude the objects from the current MDS configuration. PERMAP will recompute the solution without these points. This option enables the user to test the influence of these points on the solution. If you want to use the object again, then you can drag the object back from the parking lot to the MDS configuration. In addition, points can be moved by dragging them around in the MDS solution. It is also possible to lock certain points that will keep them at a fixed location. Figure A.3 shows a screen shot of PERMAP using the politicians data. Text labels can be attached to the points by providing them as the first entry on a line with dissimilarities. The program can compute solutions in 1 to 4 dimensions. However, for three- or four-dimensional solutions, PERMAP shows a 2D projection of the 4D space. Note that these 2D projections may differ when PERMAP reruns the analysis.

The data input for PERMAP comes from a text file that can be written with any editor. The data file must be structured by certain specific keywords that instruct PERMAP how to read the information. Other text is simply ignored and can be used to explain the data. A simple setup for our politicians data looks like this:

```
TITLE= Example setup: politicians
NOBJECTS= 5
DISSIMILARITYLIST
Humphrey  0
MacGovern 1  0
```

```
Percy     5  2  0
Wallace   7 10  9  0
Nixon     6  8  4  3  0
```

A nonexhaustive list of subcommands of PERMAP is given below.

- NOBJECTS sets the number of objects.
- TITLE and SUBTITLE set the title and subtitle to be used in the output. If the MESSAGE and SUBMESSAGE are specified, then these are used in the output.
- DISSIMILARITYLIST or SIMILARITYLIST indicate that a triangular matrix with the dissimilarities or similarities are specified below. Note that the diagonal elements should be given as well and that NA specifies a missing value. If a dissimilarity line starts with a text entry, then it is used as a label for the points.
- WEIGHTLIST announces to PERMAP that the weights are specified below.
- ATTRIBUTELIST specifies that the two-way data to be used to compute dissimilarities follow below. Note that NATTRIBUTES (indicating the number of columns of the two-way data) has to be defined before.
- LOCATIONLIST gives the initial configuration.
- STARTMDSANALYSISTYPE defines the transformations. Choose 0 for a ratio transformation, 2 for interval, and 4 for ordinal primary approach to ties. The program cannot do the secondary approach to ties.
- STARTBADNESSFUNCTION defines what MDS loss function is used. Choose 0 for Stress, 1 for Stress-1, 2 for S-Stress, and 3 for MULTISCALE.
- STARTDISTANCEFUNCTION defines the distance to be used. Choose 0 for Euclidean distances, 1 for city-block, and 2 for Minkowski.
- STARTATTRIBUTEFUNCTIONNUM defines how the two-way data should be transformed into dissimilarities. Choose 0 for one minus the cosine of the angle between the vectors defined by the columns, 1 for Euclidean distances between the rows, 2 for city-block distances between the rows, 3 for one minus Guttman's μ_2 coefficient, 4 for the Pearson correlation between the columns, 5 for the Spearman rank correlation between the columns, 6 for the proportion of different categories between the rows (to be used for nominal variables). Options 7 to 12 are used for binary variables: 7 for the Jaccard indexes, 8 for Gower/Russel/Rao, 9 for Sokal-Michener distances, 10 for Hamman, 11 for Yule, and 12 for Askin/Charles.
- STARTDIMENSIONNUM allows to specify the dimensionality of the solution between 1 and 4.

Apart from the NOBJECTS command and a command to read the data, all other commands are optional.

For more information, contact Ron B. Heady, University of Louisiana at Lafayette, U.S.A. E-mail: ron@heady.us; Internet: http://www.ucs.louisiana.edu/~rbh8900

A.2 MDS Programs with High-Resolution Graphics

Current computers are able to provide high-resolution graphics, which is particularly important for a visualization technique such as MDS. All ma-

jor statistical packages provide these high-resolution graphics. Below we discuss several MDS procedures available in commercial statistical packages and a package called NEWMDSX© that provides a shell for text-based MDS programs that produce high-resolution graphics.

ALSCAL

ALSCAL (Takane et al., 1977) is one of the current MDS modules in SPSS. ALSCAL differs from other MDS programs in minimizing S-Stress rather than Stress, thereby fitting squared distances to squared dissimilarities. As a result, in ALSCAL the large dissimilarities are much better represented than the small dissimilarities. ALSCAL is a flexible MDS program that also provides models for asymmetric data, unfolding, and three-way analyses (by the weighted or generalized Euclidean model). Many options can be combined. ALSCAL also allows coordinates to be fixed, which is especially useful for external unfolding.

ALSCAL can be started in SPSS by choosing the menu "Analyze > Scale > Multidimensional Scaling...". Using dialogue boxes, the ALSCAL options can be specified. In addition, (dis)similarity matrices can be created from rectangular data matrices. Alternatively, ALSCAL can be run through SPSS-syntax allowing for some more options. Some care has to be taken when adapting a configuration plot in ALSCAL. If you change the range of the axes or resize the plot differently for the two axes, then the horizontal units can be different from the vertical units so that the distances you see may be misleading. In addition, the default convergence criterion is far too weak and should be manually tightened to, say, .000001 or smaller.

A sample setup for an ordinal MDS analysis with ALSCAL of a 5×5 matrix of dissimilarity scores on five politicians is this:

```
TITLE 'Alscal in SPSS example setup: politicians'.
MATRIX DATA /VARIABLES Humphrey McGovern Percy Wallace Nixon
  /CONTENTS PROX /FORMAT LOWER NODIAGONAL.
BEGIN DATA
 1
 5  2
 7 10  9
 6  8  4  3
END DATA.
ALSCAL /VARIABLES Humphrey McGovern Percy Wallace Nixon
 /CRITERIA CONVERGE(0.000001) ITER(100) STRESSMIN(0.000001)
 /LEVEL ORDINAL.
```

Commands in SPSS are ended by a dot (.); subcommands start with a slash (/) and usually have one or more keywords; keywords are printed in caps.

The ALSCAL job above first formulates a TITLE. It then defines the data setup in the MATRIX DATA command and lists the proximities between BEGIN DATA and END DATA. In the MATRIX DATA command, /VARIABLES should be followed by a list of variable names,

one for each variable (object, point). This list can also be abbreviated by 'VAR1 TO VAR5' or 'OBJECT1 TO OBJECT5'. The variable names are at most eight characters long. The subcommand /CONTENTS PROX indicates that the contents is a proximity matrix. /FORMAT LOWER NODIAGONAL indicates that only the lower triangular elements of the proximities are to be read. Finally, the desired MDS model is specified in the ALSCAL command: The VARIABLES option lists the variables that are to be mapped into points; the CRITERIA option specifies a number of technical requests for the optimization; the LEVEL option requests an ordinal MDS.

Some optional subcommands of ALSCAL are:

- /SHAPE specifies the shape of the dissimilarity matrix. Valid keywords are SYMMETRIC, ASYMMETRIC, and RECTANGULAR. SHAPE = RECTANGULAR defines unfolding.
- /LEVEL indicates the allowed transformation of the dissimilarities. Default is ORDINAL, which does monotone regression with the secondary approach to ties. For the primary approach specify, ORDINAL(UNTIE). If the proximities are similarities instead of dissimilarities, you can specify ORDINAL(SIMILAR), which may be combined with UNTIE. The keyword INTERVAL indicates interval transformations. For example, INTERVAL(3) specifies polynomial regression of the order 3. RATIO excludes the intercept and followed by '(2)' indicates quadratic polynomial regression.
- /CONDITION specifies conditionality of the transformations. In three-way scaling, MATRIX indicates that for each replication a separate transformation of the proximities has to be found (default). UNCONDITIONAL specifies that there is only one transformation for all replications. ROW means that the proximities in every row may have a different transformation, which is useful for unfolding.
- /MODEL indicates which model has to be used. EUCLIDEAN indicates the ordinary Euclidean distance (default), INDSCAL specifies the individual differences (weighted) Euclidean distance model.
- /CRITERIA controls the stopping conditions of the algorithm. CONVERGENCE (.000001) causes the program to stop whenever the difference in S-Stress between subsequent iterations is less than .000001. ITER(100) sets the maximum number of iterations to 100. STRESSMIN(.0001) causes the iterations to stop whenever S-Stress is less than .0001. NEGATIVE allows negative dimension weights in the INDSCAL model. CUTOFF(0) causes negative proximities to be treated as missing (default). DIMENS(2,5) causes ALSCAL to compute a solution in 5 dimensions, then 4, 3, and 2 dimensions. Default is DIMENS(2,2).
- /PRINT specifies print options. DATA prints the proximities. INTERMED prints intermediate results, which can generate a huge amount of output. HEADER prints a summary of options specified.
- /PLOT controls the plots made by ALSCAL. Defaults are the plots for the object configuration, the weight matrix (for INDSCAL) and Shepard plots. In addition, ALL generates a transformation plot for every replication or row (depending on CONDITION) and a plot of the weighted object coordinates for every replication (when appropriate).

For more information, contact: worldwide headquarters SPSS Inc. 233 S. Wacker Drive, 11th Floor, Chicago, IL 60606-6307, U.S.A. Phone: (312) 651-3000; Fax: (312) 651-3668; Internet: http://www.spss.com

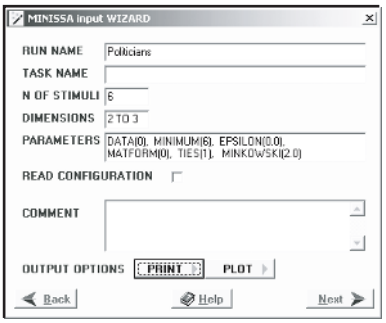

FIGURE A.4. NEWMDSX© wizard for constructing MINISSA input.

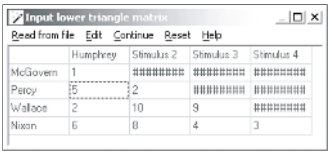

FIGURE A.5. NEWMDSX© data entry window for MINISSA.

NEWMDSX©

Many of the first-generation MDS programs have text-based input and output, and no graphical user interface nor high-resolution graphics. The package NEWMDSX© is aimed to fill this gap. It offers a shell with an easy graphical user interface to run a variety of programs reported in the literature. In this shell, you can start a wizard to construct the input file for the program you want; see Figure A.4 (based on a beta version of NEWMDSX©, version 4.0.4.). It also has several ways to read data, including a spreadsheet-like data entry window; see Figure A.5. The MDS program included in NEWMDSX© is MINISSA (discussed separately in Section A.3).

NEWMDSX© has high-resolution graphics for configurations in one to three dimensions. In addition, it provides Shepard diagrams and Stress plots. For hierarchical clustering it provides a dendrogram. The graphics the program produces can be edited.

In Figure A.6, an example is given of the graphics windows. NEWMDSX© is the only package currently available that provides a relatively easy interface for the MDS programs developed from 1960 to 1980. Table A.2 contains an overview of the MDS programs included, many of which are discussed in this book. NEWMDSX© is a package with not too many options but with a rich amount of MDS programs.

More info at: E-mail: enquiries@newmdsx.com; Internet: http://www.newmdsx.com

PROXSCAL

PROXSCAL is a program for least-squares MDS minimizing Stress available in SPSS (Commandeur & Heiser, 1993; Meulman, Heiser, & SPSS, 1999). It builds on the majorizing algorithm of De Leeuw and Heiser (1980) (see Chapter 8), which guarantees convergence of Stress. PROXSCAL offers a large variety of options for MDS analysis. One of the unique features of PROXSCAL is that the user can impose external constraints on the MDS

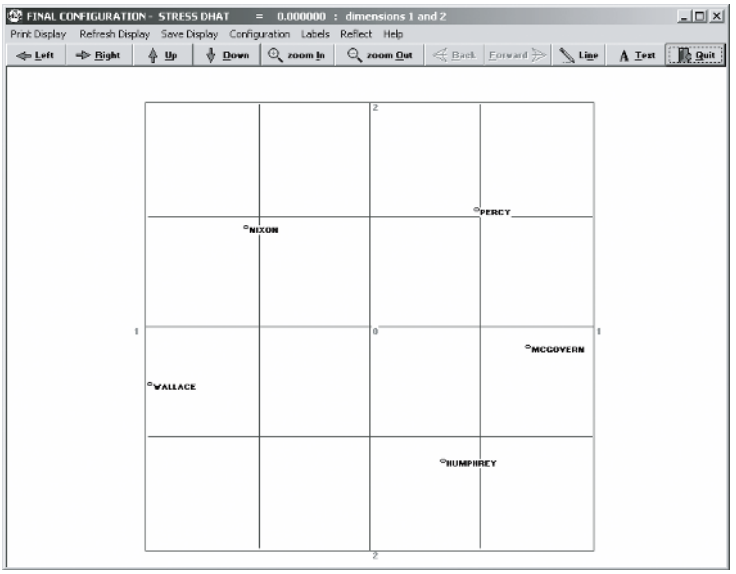

FIGURE A.6. Graphics window obtained by NEWMDSX© .

TABLE A.2. Overview of programs that can be run within NEWMDSX©.

Program	Remarks
CANDECOMP	Three-way decomposition for three-way data.
CONJOINT	Performs unidimensional conjoint analysis.
CORRESP	Correspondence analysis.
HICLUS	Performs hierarchical clustering on dissimilarity data using single or complete linkage.
INDSCAL-S	INdividual Differences SCALing for fitting the weighted Euclidean distances.
MDSORT	MDS of sorting data.
MDPREF	MultiDimensional PREFerence for the vector model of unfolding.
MINI-RSA	Ideal point unfolding model.
MINISSA	Nonmetric MDS program.
MRSCAL	MetRic SCALing for metric MDS with Minkowski distances.
PARAMAP	For maximizing local monotonicity.
PINDIS	Procrustean INdividual Differences Scaling for doing Procrustes analysis.
PREFMAP	PREFerence MAPping for external unfolding using the ideal point or vector model for unfolding.
PRO-FIT	PROperty FITting for external unfolding using the vector model.
TRISOSCAL	TRIadic Similarities Ordinal SCALing for MDS analysis of triadic dissimilarities.
WOMBATS	Work Out Measures Before Attempting To Scale converts a two-way two-mode data matrix into a dissimilarity matrix.

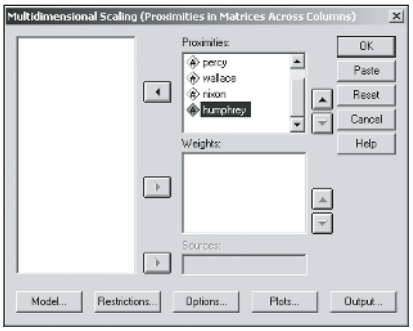

FIGURE A.7. Main dialogue boxes in SPSS PROXSCAL.

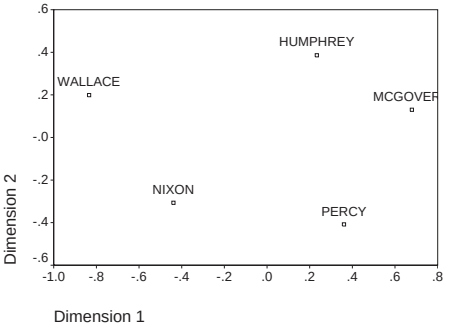

FIGURE A.8. PROXSCAL solution of the politicans data.

configuration, such as the restriction that the coordinates are a linear combination of some external variables (see Section 10.3). It also allows fixing some of the coordinates. These options can be combined with the weighted Euclidean model (Section 22.1) or the generalized Euclidean model (Section 22.2). The implementation of these models in PROXSCAL never gives negative dimension weights.

PROXSCAL can attach user-defined weights to every proximity. The transformations of the proximities in PROXSCAL are ordinal, interval, power, and monotone spline. PROXSCAL is the only program that avoids negative disparities that may arise when specifying an interval transformation of the dissimilarities as shown by Heiser (1990).

PROXSCAL can be specified in SPSS by choosing the menu "Analyze > Scale > Multidimensional Scaling (PROXSCAL)...". The main dialogue box of PROXSCAL is given in Figure A.7. Most options can be accessed through dialogue boxes. However, some options can only be specified using syntax.

```
TITLE 'Proxscal in SPSS example setup: politicians'.
MATRIX DATA /VARIABLES Humphrey McGovern Percy Wallace Nixon
  /CONTENTS MAT /FORMAT LOWER NODIAGONAL.
BEGIN DATA
 1
 5  2
 7 10  9
 6  8  4  3
END DATA.
PROXSCAL /VARIABLES Humphrey McGovern Percy Wallace Nixon
 /SHAPE = LOWER
 /INITIAL = TORGERSON
 /CRITERIA = DIMENSION(2) DIFFSTRESS(0.000001)
             MAXITER(100) MINSTRESS(0.000001)
 /TRANSFORMATION = ORDINAL.
```

PROXSCAL has many different options, some of which are only accessible from syntax. The commands in the example above up to `PROXSCAL` correspond to those discussed

above for the ALSCAL program. A nonexhaustive list of subcommands of PROXSCAL is given below.

- /VARIABLES specifies the columns of the dissimilarity matrix. Unless the /TABLE subcommand is used, the number of rows should be a multiple of the number of variables specified. In the simple case of a single dissimilarity matrix, the number of rows is equal to the number of columns. For three-way analyses, the replications should be stacked underneath each other. The labels used in the plot are either the variable names or their variable labels.
- /TABLE allows to read dissimilarities from a single column. To identify a dissimilarity, two extra variables are needed: its row and column number. If three-way data are present, a third variable is needed to define the replication (called a source in PROXSCAL). The value labels to the row, column, and replication variable are used as labels in the plots.
- /SHAPE specifies the shape of the dissimilarity matrix. Valid keywords are LOWER, UPPER, and BOTH to specify respectively the lower part of the dissimilarity matrix, the upper part, or the symmetrized table.
- /WEIGHTS is used to specify nonnegative weights for weighting the residuals. The weights are read similarly as the dissimilarities as specified by the VARIABLES subcommand.
- /PROXIMITIES defines whether the data are assumed to be DISSIMILARITIES (default) or SIMILARITIES.
- /INITIAL specifies what initial configuration is used. SIMPLEX (default) uses a PROXSCAL specific start assuming that all objects are at distance one of each other. TORGERSON defines classical scaling as a start configuration. This option tends to give better quality solutions than SIMPLEX. RANDOM(N) computes a PROXSCAL solution for `n` random starts and reports the best.
- /TRANSFORMATION indicates the allowed proximities of the dissimilarities. Default is RATIO where the dissimilarities are only multiplied such that the disparities have a specific sum of squares. The keyword INTERVAL indicates interval transformations. ORDINAL does monotone regression with the secondary approach to ties and ORDINAL(UNTIE) specifies the primary approach to ties. SPLINE(DEGREE,INKNOT) specifies a spline transformation of order `degree` and `inknot` interior knots.
- /CONDITION specifies the conditionality of the transformations. In three-way scaling, MATRIX indicates that for each replication a separate transformation of the proximities has to be found (default). UNCONDITIONAL specifies that there is only one transformation for all replications.
- /MODEL has the IDENTITY model as default thereby modeling all replications by the same configuration. For three-way data, WEIGHTED indicates the weighted Euclidean distance so that dimensions may differ in the third way according to their individual dimension weight. GENERALIZED specifies the individual differences (weighted) Euclidean distance model. REDUCED specifies the reduced rank model.
- /CRITERIA controls the stopping conditions of the algorithm. CONVERGENCE (.000001) causes the program to stop whenever the difference in S-Stress between subsequent iterations is less than .000001. MAXITER(100) sets the maximum number of iterations to 100. MINSTRESS(.0001) causes the iterations to stop whenever Stress is less than .0001. In addition, by DIMENSIONS(DMIN,DMAX) you can let PROXSCAL compute solutions from `dmax` dimensions to `dmin` dimensions.
- /PRINT specifies print options. INPUT prints the proximities, HISTORY the history of iterations, STRESS several standard Stress measures, DECOMPOSITION a decomposition of Stress into Stress per object and possibly per replication, COMMON the

coordinates, DISTANCES the distances, WEIGHTS the weights used by the generalized or weighted Euclidean distances found by three-way models, and TRANSFORMATION the transformations.

- /PLOT controls the plots made by PROXSCAL. Defaults are the plots for the object configuration (COMMON), and the weight matrix (WEIGHTS) for three-way models. In addition, a STRESS-plot is available for plotting Stress against the number of dimensions of the solution, TRANSFORMATION for plotting the the data against the disparities, RESIDUALS for plotting the disparities against the distances, and INDIVIDUAL for plotting the coordinates of replications in three-way models. Note that PROXSCAL does not provide a Shepard plot.

For more information contact: Frank M.T.A. Busing, Dept. of Psychometrics, Univ. of Leiden, P.O. Box 9555, 2300 RB Leiden, The Netherlands. E-mail: busing@fsw.leidenuniv.nl; Internet: http://www.spss.com

SAS

SAS is a comprehensive system of software products for managing, analyzing, and presenting data. SAS has versions for virtually all platforms. SAS can be run in batch mode, submit mode, interactive line mode, and display manager (windows) mode. SAS used to offer ALSCAL as its MDS module but now has replaced ALSCAL by 'PROC MDS'. This program has many options, some of them rather tricky ones, with effects that are difficult to predict.

In batch mode, our sample MDS job can be set up in the SAS command language as follows. SAS commands and options are printed in capital letters. Commands are ended with a ;.

```
DATA polit;
TITLE Politicians Example;
INPUT (var1-var5)(3.) @21 name $ 8.;
CARDS;
  0                Humphrey
  1  0             McGovern
  5  2  0          Percy
  7 10  9  0       Wallace
  6  8  4  3  0    Nixon
;
PROC MDS DIM=2 LEVEL=ORDINAL PFINAL PCONFIG OUT=OUT OUTRES=RES;
OBJECT name;
RUN;
```

The job first reads five numeric variables in fixed format (fields of length 3) and one alphanumeric variable, "name", in a field of length 8, starting in column 21. Then, the MDS procedure is called, together with a set of options. PROC MDS analyzes the proximities among all variables at the ordinal measurement level (LEVEL=ORDINAL) in two dimensions (DIMENSION=2. The fit values and the configuration are printed (PFINAL and PCONFIG, respectively). The estimates and fitted values are saved in output data files

("out" and "res", respectively). PROC MDS produces no plots. For plotting, one must utilize special plotting procedures with the output files.

SAS has its own command language. The following summarizes the most important commands for PROC MDS. The general syntax is:

```
PROC MDS  options;
   VAR  variables;
   INVAR  variables;
   OBJECT  variable;
   SUBJECT  variable;
   WEIGHT  variable;
   BY  variables;
```

The PROC MDS statement is required. All other statements are optional. The VAR statement defines the numeric variables in the file DATA = xyz that contain the proximities. Each variable corresponds to one object/point. If VAR is omitted, all numeric variables not specified in another statement are used. The INVAR statement defines the numeric variables in the file INITIAL=xyz, the initial configuration, where the first variable contains the coordinates on the first dimension, ..., the mth variable the coordinate on the mth dimension. The WEIGHT statement specifies a numeric variable in the file DATA=xyz that contains weights for each proximity. The number of WEIGHT variables must be the same as the number of VAR variables, and the variables in the WEIGHT statement must be in the same order as the corresponding variables in the VAR statement. If no WEIGHT statement is used, all data within a partition are assigned equal weight. The BY statement is used to obtain separate MDS analyses on groups of proximity data defined by the BY variables. The OBJECT statement defines a variable that contains descriptive labels for the points. The SUBJECT statement specifies a variable in the file DATA = xyz that contains descriptive labels for the data matrices or "subjects".

The options for PROC MDS are:

- The Proximities
  - DATA = SAS file name, the data set containing one or more *square* matrices to be analyzed. (The requirement to input square proximity matrices makes the procedure clumsy for unfolding applications, because off-diagonal matrices cannot be processed directly. Rather, they have to be defined as submatrices within a square matrix with missing values.) Usually, there is one matrix per person. Data are generally assumed to be dissimilarities unless (a) there are diagonal elements that are generally larger than the off-diagonal elements or (b) one uses the SIMILAR option.
  - SIMILAR causes the data to be treated as similarities.
  - SHAPE = TRIANGULAR | SQUARE determines whether the entire data matrix for each subject is stored and analyzed or only one triangle of the matrix. Default is triangle, unless CONDITION = ROW.
- The MDS Model
  - LEVEL = ABSOLUTE | RATIO | INTERVAL | LOGINTERVAL | ORDINAL specifies the admissible transformation on the proximities.
  - CONDITION = UN | MATRIX | ROW. Conditionalities of proximities. Default is MATRIX.
  - DIMENSION = $m_{\min}$ [TO $m_{\max}$], where $1 \leq m_{\min} \leq m_{\max} < n$. Skipping the TO term leads to $m_{\min} = m$.

- CUTOFF = k, causes data less than k to be treated as missing values. Default value is 0.
- UNTIE specifies the primary approach to ties for LEVEL = ORDINAL.
- NEGATIVE allows slopes or powers to be negative with LEVEL = RATIO, INTERVAL, or LOGINTERVAL.
- COEFF = IDENTITY | DIAGONAL, yields Euclidean distances and weighted Euclidean distances ("INDSCAL"), respectively.

- The Loss Function
  - FORMULA = 0 | 1 | 2 determines how the badness-of-fit criterion is standardized. 0 fits a regression model by ordinary least squares, not for level=ordinal. 1 is Stress-1 (for FIT = DISTANCE and LEVEL = ORDINAL) or S-Stress (for FIT = SQUARED and LEVEL = ORDINAL). 2 standardizes each partition specified by CONDITION; corresponds to Stress-2 for FIT = DISTANCE and LEVEL = ORDINAL. Default is 1 unless FIT = LOG.
  - ALTERNATE = NONE | MATRIX | ROW. Determines what form of alternating least-squares algorithm is used. NONE causes all parameters to be adjusted simultaneously on each iteration; best for small n (=objects) and N (=matrices). MATRIX adjusts the parameters for the first proximity matrix, then for the second, etc.; best for large N and small n. ROW adds further stages; best for large n.
  - FIT = DISTANCE | SQUARED | LOG | n specifies a fixed transformation to apply to *both* $f(p_{ij})$s and d_{ij}s before the error is computed. This leads to different weighting of large/small values. The default is DISTANCE or, equivalently, 1 which fits $f(p_{ij})$'s to d_{ij}'s. FIT = n fits nth power $f(p_{ij})$s to nth power d_{ij}s.

- Some technical options
  - MAXITER = k, the maximum number of iterations. Default is 100.
  - CONVERGENCE = k, the convergence criterion. Default is $k = .01$. Values of less than .0001 may be impossible because of machine precision.
  - RANDOM = k, causes initial coordinate values to be pseudorandom numbers with seed=k.

- Some output options
  - PFIT and PCONFIG: print the fit values and the MDS configuration, respectively. Various other print options exists, e.g., PINIT, which prints the initial values, and PTRANS, which prints the estimated transformation parameters if any are computed in metric models.
  - OUT = xyz. Creates the SAS data file "xyz" containing the estimates of all parameters of the MDS model and the value of the badness-of-fit criterion.
  - OUTRES = xyz. Creates file that contains original proximities, MDS distances, transformed proximities/distances, residuals.

For more information, contact: SAS Institute Inc., 100 SAS Campus Drive, Cary, NC 27513-2414, U.S.A. Phone: (919) 677-8000. Fax: (919) 677-4444. Internet: http://www.sas.com

STATISTICA

STATISTICA is a comprehensive package for statistics and statistical graphics that includes an MDS module. Doing MDS, STATISTICA yields windows of numerical output, high-resolution plots of the MDS configuration (also 3D configurations that can be rotated in space), fit plots such as data vs. d-hats or data vs. rank-images, and so on. The graphics windows can be modified by changing fonts, changing the line thickness, resizing points, moving point labels as objects, and the like, or by drawing into the plots with the built-in drawing tools.

STATISTICA's MDS module uses the Guttman–Lingoes initial configuration or a user-supplied external initial configuration. It then employs the MINISSA algorithm (Roskam & Lingoes, 1981), which does ordinal MDS with rank-images in the initial iterations and monotone regression later on to ensure convergence. As fit indices, Stress and Raw Stress are computed with both monotone regression values and rank-images. The coefficient of alienation is also reported.

STATISTICA's MDS only offers ordinal MDS and the Euclidean metric. Although this is sufficient for most applications, models like interval MDS, say, are needed for some data to avoid degeneracies.

STATISTICA can be run interactively (via mouse clicks that can be recorded), by submitting a program of previously stored mouse clicks, or by executing a file of STATISTICA's SCL command language. In SCL, our politicians example is set up as shown below, assuming that a system file "proxpol.sta" has been created beforehand. There exists only one further option: ITERATIONS $= k$.

```
FILE = "c:\proxpol.sta"
MDS
 / VARIABLES = ALL
 / DIMENSIONS = 2
```

For more information, contact: StatSoft Inc., 2300 East 14th St., Tulsa, OK 74104, U.S.A. Phone: (918) 749-1119. Fax: (918) 749-2217. E-mail: info@statsoft.com. Internet: http://www.statsoft.com

SYSTAT

SYSTAT is a comprehensive package for statistics, including graphics (Wilkinson & Hill, 1994). SYSTAT can be run in batch mode, submit mode, interactive line mode, and in a pull-down menu mode. As is true for all statistics packages, it is best to first set up a system file containing the proximity matrix. System files are defined via a spreadsheet, reading data from an ASCII file, or by computing proximities internally from other data. However, data can also be input from within a command file. To do a SYSTAT MDS analysis, one calls, from within the MDS module, the system file "polit.syd", say,

by typing "USE polit" (return), followed by "MODEL var1..var5" (return) and "ESTIMATE" (return). Alternatively, one first defines a command file containing these three lines and then submits this command file. (The first three letters of the commands are sufficient.) This will do an ordinal MDS in 2D, using Euclidean distances. A more explicit batch mode command job is this:

```
BASIC
SAVE polit
TYPE=DISSIMILARITY
INPUT Humphrey McGovern Percy Wallace Nixon
DIAGONAL=ABSENT
RUN
1
5  2
7 10  9
6  8  4  3
~
MDS
USE polit
MODEL Humphrey..Nixon
ESTIMATE / LOSS=KRUSKAL, REG=MONOTONIC, DIM=2, R=2
```

If the batch job is called "mds.cmd", it is run by typing "systat < mds.cmd" from the DOS prompt. The resulting configuration and its Shepard diagram are shown on the computer screen, where they can be fine-tuned, previewed, and sent to a printer. The results can also be saved, along with distances, d-hats, and residuals for further analyses. In combination with a built-in spreadsheet editor, the points in the configuration plot can be labeled. Using this feature, one can, for example, create a facet diagram, where the points are labeled by their codings on a particular facet (see Chapter 5). Three-dimensional plots with embedded axes, labels, surfaces, and the like, are also available.

For an experienced data analyst, SYSTAT is best used with its command language and submit files. All SYSTAT jobs can be documented and easily modified if needed. However, even using pull-down menus or Windows, command language files are automatically generated for previewing and saving.

A SYSTAT MDS job generally looks like this:

```
MDS
 MODEL variables / options
 CONFIG arguments
 SAVE filename / arguments
 ESTIMATE / options
```

- The MODEL options: ROWS = N and SHAPE = RECT | SQUARE: when doing unfolding, one needs to specify that the proximity matrix is "rectangular" with N rows; the number of columns corresponds to the number of objects n.

- The CONFIG arguments: CONFIG = [coordinates of first point; coordinates of second point; ...] or CONFIG = LAST. The LAST argument allows using the configuration from the previous scaling. An example for an external 3-point configuration in 2D is CONFIG = [1 2; 3.3 5; 4 5].
- The SAVE arguments: Specifying DISTANCES saves the MDS distances; CONFIG saves the final MDS configuration; RESID saves the proximities, distances, d-hats, and residuals, together with row and column subscripts.
- The ESTIMATE options:
  - DIMENSION = m specifies the dimensionality m of the MDS solution.
  - REGRESSION = MONOTONIC | LINEAR | LOG | POWER uses ordinal, interval, log-interval, or power-function MDS, respectively.
  - SPLIT = ROW is split-by-rows (unfolding); SPLIT =MATRIX is split-by-matrix conditionality for stacked proximity matrices.
  - WEIGHT is individual differences scaling with dimension weights for each matrix.
  - R = r specifies the exponent of the Minkowski metric. For example, R = 2 requests Euclidean distances.
  - LOSS = KRUSKAL | GUTTMAN | YOUNG specifies the loss function. KRUSKAL is Stress-1; GUTTMAN is the coefficient of alienation, K; YOUNG is S-Stress.
  - ITERATIONS = k sets the maximum number of iterations to k.
  - CONVERGE = k causes MDS to stop when the maximum absolute difference between any coordinate in the solution $\mathbf{X}$ at iteration i and iteration $i + 1$ is less than k.

The program defaults are DIM =2, REGR = MONO, no split, no weight, LOSS = KRUS, R = 2, ITER = 50, DECREMENT = 0.005.

For more information, contact: Systat Software, Inc.; 501 Canal Blvd; Suite E; Point Richmond, CA 94804-2028; U.S.A. Phone: (800) 797-7401. Fax: (800) 797-7406. E-mail: info-usa@systat.com. Internet: http://www.systat.com

A.3 MDS Programs without High-Resolution Graphics

Most programs from the early days of MDS lack high-resolution graphics. Nevertheless, these programs are usually well documented in the literature and often used in simulation studies. For completeness, we discuss a few of these programs.

The Guttman–Lingoes Nonmetric PC Series

The GL Series is a collection of 32 individual programs for the analysis of qualitative and ordinal data. The philosophy of the GL Series is to have compact programs that are good for particular purposes rather

than one jumbo program that does everything. For MDS, the main program is MINISSA-I for unconditional and row/column conditional proximities, but there are also special programs for proximities that are both row and column conditional at the same time (SSA-II), for imposing a number of independent sets of order constraints onto the distances (CMDA), for individual differences scaling (PINDIS), or for representing proximities in a vector model (SSA-III). A complete documentation of the source code (FORTRAN IV) of most of the programs is available (Lingoes, 1973).

The GL programs are essentially batch programs. The user is required to "punch" his or her specifications as numbers in four-field columns of the parameter "cards" of the batch job (see below, fourth row of batch job). A typical set-up for MINISSA-I for our politicians data looks as follows.

```
SSA-I.OUT (name of file for numerical output)
SSA-I.PLT (name of file for printer-plot output)
1 MDS of five politicians ("title card")
   5   2   2   0   0   1   0   0   0   0   0   0   0   0   0
(5F3.0)
  1
  5  2
  7 10  9
  6  8  4  3
```

The various numerical entries on the fourth "card" specify the number of objects (5), the lowest MDS dimensionality (2), the highest MDS dimensionality (2), the type of proximities (0=dissimilarities), a request to print out the distance matrix (0=no), and a request to minimize Kruskal's Stress (1=yes). The remaining zeros refer to the usual defaults: Adding points to a fixed configuration (no=0, yes=1), external initial configuration (no=0, yes=1), special switch for program options (0=no special setting,1=ignore cells in SSA-I/MDS, number of column fields for SSAR-I/unfolding), ignore cells in data matrix (no=0, yes=1), missing data (no=0, yes=1), distance formula (Euclidean=0, city-block=1), differential weighting of small and large distances (global weighting=0, local weighting=1), type of analysis (SSA-I/MDS=0, SSAR-I/unfolding=1), missing data code (real value), value above/below which all input data are considered tied (real value).

Most of the GL programs can be downloaded from http://www.newmdsx.com

FSSA: *Faceted Smallest Space Analysis*

A special MDS program is FSSA by Shye (1991). FSSA is a stand-alone program that is public domain for scientists. It analyzes from 3 to 50 objects in two to ten dimensions, using the Guttman algorithm. What makes FSSA unique is that it allows one to code the objects with respect to several facets (see Chapter 5). FSSA then partitions the resulting 2D planes, facet by facet, in three ways (axial, polar, modular), using parallel straight lines, concentric circles, and rays emanating from a common origin, respectively. The partitionings are shown as screen diagrams.

The program can be obtained from: Samuel Shye, Dept. of Psychology, Hebrew University of Jerusalem, Jerusalem 91905, Israel. E-mail: msshye@pluto.mscc.huji.ac.il

KYST

KYST (Kruskal et al., 1978) is an exceedingly flexible MDS program. It allows one to set up virtually any MDS model, except MDS with side constraints on coordinates or distances. KYST is noncommercial software, available on the Internet at http://www.netlib.org/mds/. The program is written in FORTRAN and can, in principle, be compiled on any machine. KYST allows the user to specify Minkowski distances other than the Euclidean distance. Moreover, weights can be assigned to each proximity, for example, for weighting each datum by its reliability. Another feature is the possibility of polynomial regression for specifying the transformation function of the proximities. Because KYST is an old program, it provides only printer-type graphics with 132 characters per line. KYST's manual is not made for today's occasional user of MDS, but the logic of KYST's command language is straightforward.

An example setup for a KYST2e job is the following.

```
DIMMAX = 2, DIMMIN = 2
REGRESSION = ASCENDING
COORDINATES = ROTATE
ITERATIONS = 100
PRINT = HISTORY
PRINT = DATA
PLOT = SCATTER = ALL
PLOT = CONFIGURATION
TORSCA
DATA, LOWERHALFMATRIX, DIAGON = ABSENT
KYST example setup: politicians
  5  1  1
(5F3.0)
  1
  5  2
  7 10  9
  6  8  4  3
COMPUTE
STOP
```

The setup consists of control lines and the data definition lines (data deck). The order of the control lines mostly does not matter. The data definition lines start in this example with the line "DATA, LOWERHALFMATRIX,..." and end with the line " 6 8 4 3". These lines are in strict order and may not be reordered. Also, the REGRESSION control lines should precede the data definition lines. Some of the control commands on the control lines are:

- Analysis options: DIMMAX = 3 sets the maximum dimensionality to 3, DIMMIN = 2 sets the minimum dimensionality to 2. SFORM1 makes the program use Kruskal's Stress formula 1, SFORM2 requests Kruskal's Stress formula 2. PRIMARY requests

the primary approach to ties, and SECONDARY the secondary approach. ITERATIONS = 100 sets the maximum number of iterations to 100, SRATST = 0.9999 causes KYST to stop the iterations whenever the ratio of subsequent Stress values is smaller than 0.9999. STRMIN = .00001 stops the iterations if the Stress becomes smaller than .00001. TORSCA takes the classical scaling solution as the initial configuration. COORDINATES = ROTATION causes the solution to be rotated to principal axes (recommended).

- REGRESSION specifies the type of admissible transformation of the proximities. REGRESSION = ASCENDING indicates that the proximities are dissimilarities, REGRESSION = DESCENDING that they are similarities. REGRESSION = POLYNOMIAL = 3 specifies a third-degree polynomial regression transformation. Adding REGRESSION = CONSTANT (or NOCONSTANT) makes the program pick an optimal additive constant. Thus, an interval transformation is specified by REGRESSION = POLYNOMIAL = 1, REGRESSION = CONSTANT.
- Print and plot options. PRINT = (NO)DATA prints the proximities, PRINT = (NO)HISTORY outputs the history of iterations, and PRINT = (NO)DISTANCES prints the distances and the disparities. When a range of dimensions is specified, a plot of Stress against the number of dimensions is given. PLOT = SCATTER = ALL (or = NONE for no plots) produces a plot of distances and disparities against the proximities. PLOT = CONFIGURATION = ALL (or =SOME or = NONE) causes KYST to plot all principal components projection planes of the MDS configuration. SOME only plots the first principal component against all other components.
- Data definition. The data are defined by five parts: (1) a line ("card") specifying the data; (2) a title line; (3) a line of parameters; (4) a line with the format of the data; (5) lines specifying the data. Line 1 of the data definition part starts with DATA followed by LOWERHALFMATRIX to indicate the lower triangular elements, UPPERHALFMATRIX indicates the upper triangular elements, and MATRIX specifies the full matrix. For triangular matrices, one specifies either DIAGONAL = PRESENT or = ABSENT. For unfolding, one sets LOWERCORNERMATRIX (or UPPERCORNERMATRIX). Line 3 expects 3 or 4 numbers that should end in columns 3, 6, 9, and 12, respectively. The parameters specify number of objects, number of replications (usually 1), number of groups (usually 1), respectively. For corner matrices, the parameters are: number of rows; number of columns; the number of replications; number of groups. Line 4 contains a FORTRAN format, for example, (5F10.3). Line 5 (and further) contains the data, possibly followed by weights. The weights definition block is specified in the same way as the data definition block, except that the word DATA has to be replaced by WEIGHTS.
- COMPUTE causes the program to start computing. After the COMPUTE command, further MDS jobs can be set up, before a final STOP.

For more information contact: Scott M. Smith, Ph.D., Dept. of Marketing, 634 TNRB Brigham Young University, Provo, Utah 84602, U.S.A. E-mail: smsmith@byu.edu. Phone: (801) 376-1339. Fax: (801) 705-9430. Internet: http://marketing.byu.edu

MULTISCALE

MULTISCALE (Ramsay, 1977) is one of the few MDS programs that offers a Maximum Likelihood (ML) approach. For ML, it can assume normal

or lognormal distributions of the residuals and assumes that the errors are independent. MULTISCALE is one of the few programs that yields confidence regions for each point. The program has various options, including, for example, spline transformations, power transformations, and the weighted Euclidean model. The output of this program is somewhat hard to read because it contains much statistical information. However, the program has a clearly written manual. The MS-DOS version of MULTISCALE has high-resolution graphics and can output postscript plots. MULTISCALE runs on several operating systems.

A sample setup for MULTISCALE is

```
@TITLE LINES=2;
  Example setup of {\sc Multiscale}:
  similarity of 5 politicians,
@PARAMETERS NSTIM=5, NSUB=1, NDIM=2,
            TRAN=SPLINES,  DISTRIB=LOG;
@DISDATA VECTOR FORMAT=FREE;
1
5 2
7 10 9
6 8 4 3
@STIMLABS FORMAT=FREE;
Humphrey McGovern Percy Wallace Nixon
@COMPUTE;
```

MULTISCALE has its own command language. The following summarizes the most important commands. The input of MULTISCALE is organized in blocks. Each block starts with an @ and ends with a semicolon (;). For example, there exists a block for the title, a block for the parameters of the analysis, and a block for reading the data. For the most part, these blocks can appear in any order, with the following exceptions: the PARAMETER block must be the first block or the second block just after the TITLE block, and the COMPUTE block is the last block before the analysis takes place. Several runs can be specified in one file by repeating a (possibly different) PARAMETER (and TITLE) block ended by the next COMPUTE block.

Here is a short description of some of the blocks:

- The TITLE block indicates that the next line is a title. LINES = 2 specifies that the title has two lines (maximum is five lines).
- The PARAMETER block sets all the important options of the analysis. NSTIMULI defines the number of objects, NDIMENSIONS sets the number of dimensions, NSUBJECTS specifies the number of replications, NKNOTS sets the number of interior knots when using spline transformation, PROBABILITY sets the confidence level for the confidence regions for points, METRIC = IDENTITY (default) for weighting every dimension equal and METRIC = DIAGONAL for weighting each dimension separate (needed for the weighted Euclidean model). TRANSFORM sets the transformation of the proximities: the keyword SCALE sets the disparity (optimally) to the sum of the proximity and an additive constant only, POWER assumes interval level of the proximities, SPLINE specifies monotone splines. With DISTRIBUTION = LOGNORMAL, we assume that the error distribution is lognormal (default); with NORMAL, we simply obtain the Stress function. DCOMPLETE indicates that the complete proximity matrix is available for each replication instead of the lower triangular elements, LISTDATA prints input matrices, NOSTATS avoids printing statistics

for replications, NOASYMPT avoids printing asymptotic variance estimates, NODIST suppresses the matrix of distances, and TABLES prints the matrix of disparities, distances and normalized residuals for each replication. DPLOT plots the disparities against the distances, TPLOT plots the disparities against the proximities, and QPLOT plots the normalized residuals against quantiles of the normal distribution. For preference data, specify PCOMPLETE for complete preference matrix instead of lower triangular elements, PPLOT for plotting the preference against predicted preference.

- The DISDATA block inputs the proximities. MULTISCALE assumes that the proximities are entered in lower triangular format. However, VECTOR indicates that the proximities are in one large vector, not in lower triangular format. DIAGONAL specifies that diagonal values are present. FORMAT = fixed specifies that the data are read by a FORTRAN format, which is entered before the first line of data. FORMAT = FREE lets MULTISCALE read the data in free format, which means that the proximities should be separated by a space or a comma.
- The STIMLABELS block allows you to input labels of the objects. The labels should be on the lines following this block. By specifying FORMAT=FREE, the labels have to be separated by a space or a comma. In the same way, the SUBLABELS block allows you to specify labels for the replications.
- The COMPUTE block starts the analysis. ITMAX=100 sets the maximum number of iterations to 100, and CONV=.005 sets a convergence criterion dependent on the log-likelihood.

The program can be obtained free of charge from: James O. Ramsay, Dept. of Psychology, McGill University, 1205 Docteur Penfield Avenue, Montreal, Québec H4A 1B1, Canada. E-mail: ramsay@psych.mcgill.ca, Internet: ftp://ego.psych.mcgill.ca/pub/ramsay/multiscl/

Appendix B
Notation

For convenience, we summarize the notation used throughout this book. We use the following conventions: a lowercase italic character denotes a scalar, a lowercase bold character denotes a vector, and a uppercase bold character denotes a matrix. Elements of vectors or matrices are denoted by a subscripted scalar. A function is usually denoted by a character followed by an argument in parentheses, for example, $f(\mathbf{x})$ is a scalar function of the vector $\mathbf{x}$, and $\mathbf{A}(\mathbf{x})$ is a matrix function of the vector $\mathbf{x}$. Some explicit notation follows below.

n	Number of objects, persons, and so on.
i, j	Running index for objects, $i, j = 1, \ldots, n$.
m	Number of dimensions.
a	Running index for dimensions, $a = 1, \ldots, m$.
$\mathbf{X}$	Matrix of coordinates x_{ia} of n objects on m dimensions.
p_{ij}	Proximity between object i and j. It could be either a similarity or a dissimilarity measure.
δ_{ij}	Nonnegative dissimilarity between object i and j.
$\boldsymbol{\Delta}$	Symmetric matrix of nonnegative dissimilarities δ_{ij} of size $n \times n$, with $\delta_{ii} = 0$.

$d_{ij}(\mathbf{X})$ The Euclidean distance distance between row i and row j of $\mathbf{X}$; that is, $d_{ij}^2(\mathbf{X}) = \sum_{a=1}^{m} (x_{ia} - x_{ja})^2$.

d_{ij} A shorter notation for the Euclidean distance $d_{ij}(\mathbf{X})$.

$\widehat{d}_{ij}$ Disparity between objects i and j. Disparities are admissibly transformed proximities that optimally approximate given distances.

w_{ij} A nonnegative weight used to (down)weight the residual in the Stress function.

$\mathbf{W}$ Symmetric matrix of weights w_{ij} with zero diagonal.

$\mathbf{D}(\mathbf{X})$ Matrix of Euclidean distances between the rows of $\mathbf{X}$.

$\mathbf{A}'$ The transpose of $\mathbf{A}$.

$\mathbf{I}$ The identity matrix, which is a square matrix with diagonal elements equal to 1 and off-diagonal elements equal to 0.

$\mathbf{1}$ A column vector with all elements equal to 1.

$\mathbf{J}$ The $n \times n$ centering matrix, $\mathbf{J} = \mathbf{I} - n^{-1}\mathbf{1}\mathbf{1}'$, where all elements of the matrix $\mathbf{1}\mathbf{1}'$ are equal to 1.

$\mathbf{A}^q$ The qth power of a square matrix $\mathbf{A}$. For example, $\mathbf{A}^3 = \mathbf{A}\mathbf{A}\mathbf{A}$.

$\mathbf{A}^{-1}$ The matrix inverse of a square matrix $\mathbf{A}$ assuming that $\mathbf{A}$ is of full rank, so that $\mathbf{A}^{-1}\mathbf{A} = \mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$.

$\mathbf{A}^-$ A generalized inverse of a square matrix $\mathbf{A}$ where $\mathbf{A}$ may be rank deficient, so that $\mathbf{A}\mathbf{A}^-\mathbf{A} = \mathbf{A}$ and $\mathbf{A}^-\mathbf{A}\mathbf{A}^- = \mathbf{A}^-$ holds. Usually, we choose the Moore–Penrose inverse $\mathbf{A}^+$ as the generalized inverse.

$\mathbf{A}^{(2)}$ Matrix $\mathbf{A}$ with squared elements.

$\mathbf{A}(\mathbf{X})$ Any matrix function of $\mathbf{X}$ with elements $a_{ij}(\mathbf{X})$.

tr $\mathbf{A}$ The trace operator sums the diagonal elements of $\mathbf{A}$; that is, tr $\mathbf{A} = \sum_{i=1}^{n} a_{ii}$.

$\widehat{\phi}(\mathbf{x}, \mathbf{y})$ Majorizing function of $\phi(\mathbf{x})$ for which $\phi(\mathbf{x}) \leq \widehat{\phi}(\mathbf{x}, \mathbf{y})$ and $\phi(\mathbf{x}) = \widehat{\phi}(\mathbf{x}, \mathbf{x})$ holds for all feasible $\mathbf{x}$ and $\mathbf{y}$.

$\|\mathbf{X}\|$ The Euclidean norm of matrix $\mathbf{X}$; that is, $\|\mathbf{X}\|^2 = \sum_{i=1}^{n} \sum_{a=1}^{m} x_{ia}^2$.

$\|\mathbf{X}\|_{\mathbf{V}}$ The weighted Euclidean norm of matrix $\mathbf{X}$; that is, $\|\mathbf{X}\|_{\mathbf{V}}^2 = \text{tr } \mathbf{X}'\mathbf{V}\mathbf{X}$.

σ_r Raw Stress, that is, the sum of the squared differences between the optimally transformed proximities $f(p_{ij})$ (i.e., the disparities $\widehat{d}_{ij}$) and the corresponding distances $d_{ij}(\mathbf{X})$ of the MDS configuration $\mathbf{X}$.

σ_n Normalized Stress, that is, raw Stress divided by the sum of squared dissimilarities or disparities.

σ_1 Stress formula 1, that is, the square root of raw Stress divided by the sum of squared distances.

σ_2 Stress formula 2, that is, the square root of raw Stress divided by the sum of squares of the distances minus the average distance.

References

Abelson, R. P., & Sermat, V. (1962). Multidimensional scaling of facial expressions. *Journal of Experimental Psychology*, *63*, 546–554.

Ahrens, H. J. (1972). Zur Verwendung des Metrikparameters multidimensionaler Skalierungen bei der Analyse von Wahrnehmungsstrukturen. *Zeitschrift für experimentelle und angewandte Psychologie*, *19*, 173–195.

Ahrens, H. J. (1974). *Multidimensionale Skalierung.* Weinheim, Germany: Beltz.

Andrews, F. M., & Inglehart, R. F. (1979). The structure of subjective well-being in nine Western societies. *Social Indicators Research*, *6*, 73–90.

Arabie, P. (1991). Was Euclid an unnecessarily sophisticated psychologist? *Psychometrika*, *56*, 567–587.

Attneave, F. (1950). Dimensions of similarity. *American Journal of Psychology*, *3*, 515–556.

Bagozzi, R. P. (1993). Assessing construct validity in personality research: Applications to measures of self-esteem. *Journal of Research in Personality*, *27*, 49–87.

Bailey, R. A., & Gower, J. C. (1990). Approximating a symmetric matrix. *Psychometrika*, *55*, 665–675.

Baird, J. C., & Noma, E. (1978). *Fundamentals of scaling and psychophysics.* New York: Wiley.

Barnes, S. H., Kaase, M., Allerback, K. R., Farah, B., Heunks, F., Inglehart, R., Jennings, M. K., Klingemann, H. D., Marsh, A., & Rosenmayr, L. (Eds.). (1979). *Political action: Mass participation in five western democracies.* Beverly Hills, CA: Sage.

Beaton, A. E., & Tukey, J. (1974). The fitting of power series, meaningful polynomials, illustrated on band-spectroscopic data. *Technometrics*, *16*, 147–185.

Bell, D. R., & Lattin, J. M. (1998). Shopping behavior and consumer preference for store price format: Why 'large basket' shoppers prefer EDLP. *Marketing Science*, *17*, 66–88.

Bendixen, M. (1996). A practical guide to the use of correspondence analysis in marketing research. *Marketing Research On-Line*, *1*, 16–38.

Bentler, P. M., & Weeks, D. G. (1978). Restricted multidimensional scaling models. *Journal of Mathematical Psychology*, *17*, 138–151.

Benzécri, J. P., et al.. (1973). *L'Analyse des données.* Paris: Dunod.

Beuhring, T., & Cudeck, R. (1985). *Development of the culture fair interest inventory: Experimental junior version* (Tech. Rep. No. Final Report on Project No. 85/1). Pretoria, South Africa: Human Sciences Research Council.

Bijleveld, C. C. J. H., & De Leeuw, J. (1991). Fitting longitudinal reduced-rank regression models by alternating least squares. *Psychometrika*, *56*, 433–447.

Bijmolt, T. H. A., & Wedel, M. (1995). The effects of alternative methods of collecting similarity data for multidimensional scaling. *Research in Marketing*, *12*, 363–371.

Bilsky, W., Borg, I., & Wetzels, P. (1994). Assessing conflict tactics in close relationships: A reanlysis of a research instrument. In J. J. Hox, P. G. Swanborn, & G. J. Mellenbergh (Eds.), *Facet theory: Analysis and design* (pp. 39–46). Zeist, The Netherlands: Setos.

Blasius, J., & Greenacre, M. J. (1998). *Visualization of categorical data.* San Diego: Academic.

Bloxom, B. (1968). *Individual differences in multidimensional scaling* (Tech. Rep. No. ETS RM 68-45). Princeton, NJ: Educational Testing Service.

Bloxom, B. (1978). Constrained multidimensional scaling in n spaces. *Psychometrika*, *43*, 397–408.

Blurton Jones, N. G. (1968). Observations and experiments on causation of threat displays of the great tit (parus major). *Animal Behaviour Monographs*, *27*, 74–158.

Boender, C. G. E. (1984). *The generalized multinomial distribution: A Bayesian analysis and applications.* Unpublished doctoral dissertation, Erasmus University, Rotterdam.

Borg, I. (1977a). Some basic concepts of facet theory. In J. C. Lingoes, E. E. Roskam, & I. Borg (Eds.), *Geometric representations of individual differences.* Ann Arbor, Michigan: Mathesis.

Borg, I. (1977b). SFIT: Matrix fitting when points correspond in some substantive sense only. *Journal of Marketing Research*, *14*, 556–558.

Borg, I. (1978a). Einige metrische und nichtmetrische Untersuchungen über den Gesamtzusammenhang der Items im ABB. In O. Neuberger & M. Allerbeck (Eds.), *Messung und Analyse von Arbeitszufriedenheit.* Bern, Switzerland: Huber.

Borg, I. (1978b). PAL: Point-wise alienation coefficients in multidimensional scaling. *Journal of Marketing Research*, *15*, 478–479.

Borg, I. (1979). Ein Verfahren zur Analyse metrischer asymmetrischer Proximitätsmatrizen. *Archiv für Psychologie*, *131*, 183–194.

Borg, I. (1988). Revisiting Thurstone's and Coombs' scales on the seriousness of crimes and offences. *European Journal of Social Psychology*, *18*, 53–61.

Borg, I. (1999). MTMM, FT, MDS, and FA. In R. Meyer Schweizer (Ed.), *Proceedings of the seventh facet theory conference* (pp. 3–18). Bern, Switzerland: Bern University.

Borg, I., & Bergermaier, R. (1981). Some comments on The structure of subjective wellbeing in nine Western societies by Andrews and Inglehart. *Social Indicators Research*, *9*, 265–278.

Borg, I., & Bergermaier, R. (1982). Degenerationsprobleme im Unfolding und ihre Lösung. *Zeitschrift für Sozialpsychologie*, *13*, 287–299.

Borg, I., & Groenen, P. J. F. (1995). Asymmetries in multidimensional scaling. In F. Faulbaum (Ed.), *Softstat '95* (pp. 31–35). Stuttgart: Gustav Fischer.

Borg, I., & Groenen, P. J. F. (1997). Multitrait-multimethod by multidimensional scaling. In F. Faulbaum & W. Bandilla (Eds.), *Softstat '97* (pp. 59–66). Stuttgart: Lucius.

Borg, I., & Groenen, P. J. F. (1998). Regional interpretations in multidimensional scaling. In J. Blasius & M. Greenacre (Eds.), *Visualizing categorical data.* New York: Academic.

Borg, I., & Leutner, D. (1983). Dimensional models for the perception of rectangles. *Perception and Psychophysics*, *34*, 257–269.

Borg, I., & Leutner, D. (1985). Measuring the similarity of MDS configurations. *Multivariate Behavioral Research*, *20*, 325–334.

Borg, I., & Lingoes, J. (1987). *Multidimensional similarity structure analysis.* Berlin: Springer.

Borg, I., & Lingoes, J. C. (1977). Geometric representations of individual differences. In J. C. Lingoes (Ed.), *Geometric representations of relational data* (1st ed., pp. 216–284). Ann Arbor, MI: Mathesis.

Borg, I., & Lingoes, J. C. (1980). A model and algorithm for multidimensional scaling with external constraints on the distances. *Psychometrika*, *45*, 25–38.

Borg, I., Schönemann, P. H., & Leutner, D. (1982). Merkmalsüberdeterminierung und andere Artefakte bei der Wahrnehmung einfacher geometrischer Figuren. *Zeitschrift für experimentelle und angewandte Psychologie*, *24*, 531–544.

Borg, I., & Shye, S. (1995). *Facet theory: Form and content.* Newbury Park, CA: Sage.

Borg, I., & Staufenbiel, T. (1984). Zur Beziehung von optimalem Stress-Wert und richtiger Minkowski-Metrik. *Zeitschrift für experimentelle und angewandte Psychologie*, *31*, 376–390.

Borg, I., & Staufenbiel, T. (1986). The MBR metric. *Journal of Mathematical Psychology*, *30*, 81—84.

Borg, I., & Staufenbiel, T. (1993). Facet theory and design for attitude measurement and its application. In D. Krebs & P. Schmidt (Eds.), *New directions in attitude measurement* (pp. 206–237). New York: De Gruyter.

Borg, I., & Tremmel, L. (1988). Compression effects in pairwise ratio scaling. *Methodika*, *2*, 1–15.

Bortz, J. (1974). Kritische Bemerkungen über den Einsatz nichteuklidischer Metriken im Rahmen der multidimensionalen Skalierung. *Archiv für Psychologie*, *126*, 194–212.

Bove, G., & Critchley, F. (1993). Metric multidimensional scaling for asymmetric proximities when the asymmetry is one-dimensional. In R. Steyer, K. F. Wender, & K. F. Widamann (Eds.), *Pcychometric methodology: Proceedings of the 7th European Meeting of the Psychometric Society in Trier* (pp. 55–60). Stuttgart: Gustav Fischer Verlag.

Broderson, U. (1968). *Intra- und interindividuelle mehrdimensionale Skalierung eines nach objektiven Kriterien variierenden Reizmaterials.* Unpublished master's thesis, Christian-Albrechts-Universität.

Brokken, F. B. (1983). Orthogonal Procrustes rotation maximizing congruence. *Psychometrika*, *48*, 343–352.

Browne, M. W. (1969). On oblique procrustes rotations. *Psychometrika*, *34*, 375–394.

Browne, M. W. (1972a). Oblique rotation to a partially specified target. *British Journal of Mathematical and Statistical Psychology*, *25*, 207–212.

Browne, M. W. (1972b). Orthogonal rotation to a partially specified target. *British Journal of Mathematical and Statistical Psychology*, *25*, 115–120.

Browne, M. W. (1987). The Young-Housholder algorithm and the least squares multidimensional scaling of squared distances. *Journal of Classification*, *4*, 175–190.

Browne, M. W., & Kristof, W. (1969). On the oblique rotation of a factor matrix to a specified pattern. *Psychometrika*, *34*, 237–248.

Brusco, M. J. (2001). A simulation annealing heuristic for unidimensional and multidimensional (city-block) scaling of symmetric proximity matrices. *Journal of Classification*, *18*, 3–33.

Brusco, M. J. (2002). Integer programming methods for seriation and unidimensional scaling of proximity matrices: A review and some extensions. *Journal of Classification*, *19*, 45–67.

Brusco, M. J., & Stahl, S. (2000). Using quadratic assignment methods to generate initial permutations for least quares unidimensional scaling of symmetric proximity matrices. *Journal of Classification*, *17*, 197–223.

Buja, A., Logan, B. F., Reeds, J. R., & Shepp, L. A. (1994). Inequalities and positive-definite functions arising from a problem in multidimensional scaling. *The Annals of Statistics*, *22*, 406–438.

Buja, A., & Swayne, D. F. (2002). Visualization methodology for multidimensional scaling. *Journal of Classification*, *19*, 7–44.

Burton, M. L. (1975). Dissimilarity measures for unconstained sorting data. *Multivariate Behavioral Research*, *10*, 409–424.

Busing, F. M. T. A. (2004). Personal communication.

Busing, F. M. T. A. (2005). Avoiding degeneracy in metric unfolding by penalizing the intercept. (Unpublished manuscript)

Busing, F. M. T. A., Groenen, P. J. F., & Heiser, W. J. (2005). Avoiding degeneracy in multidimensional unfolding by penalizing on the coefficient of variation. *Psychometrika*, *70*, 71–98.

Cailliez, F. (1983). The analytical solution to the additive constant problem. *Psychometrika*, *48*, 305–308.

Carroll, J. D. (1972). Individual differences and multidimensional scaling. In R. N. Shepard, A. K. Romney, & S. B. Nerlove (Eds.), *Geometric representations of individual differences.* New York: Academic.

Carroll, J. D. (1980). Models and methods for multidimensional analysis of preferential choice or other dominance data. In E. D. Lantermann & H. Feger (Eds.), *Similarity and choice.* Bern, Switzerland: Huber.

Carroll, J. D., & Arabie, P. (1980). Multidimensional scaling. *Annual Review of Psychology*, *31*, 607–649.

Carroll, J. D., & Chang, J. J. (1970). Analysis of individual differences in multidimensional scaling via an N-way generalization of 'Eckart-Young' decomposition. *Psychometrika*, *35*, 283–320.

Carroll, J. D., & Chang, J. J. (1972, March). *IDIOSCAL (Individual Differences In Orientation SCALing): A generalization of INDSCAL allowing idiosyncratic reference systems as well as an analytic approximation to INDSCAL.* Paper presented at the spring meeting of the Psychometric Society. Princeton, NJ.

Carroll, J. D., Green, P. E., & Carmone, F. J. (1976). *CANDELINC: A new method for multidimensional scaling with constrained solutions.* Paper presented at the International Congress of Psychology. Paris.

Carroll, J. D., & Wish, M. (1974a). Models and methods for three-way multidimensional scaling. In D. H. Krantz, R. C. Atkinson, R. D. Luce, & P. Suppes (Eds.), *Contemporary developments in mathematical psychology* (Vol. II, pp. 57–105). San Francisco: Freeman.

Carroll, J. D., & Wish, M. (1974b). Multidimensional perceptual models and measurement methods. In E. C. Carterette & M. P. Friedman (Eds.), *Handbook of perception* (Vol. II, pp. 391–447). New York: Academic.

Chang, J. J., & Carroll, J. D. (1969). *How to use MDPREF, a computer program for multidimensional analysis of preference data.* Computer Manual. Murray Hill, NJ: Bell Labs.

Chaturvedi, A., & Carroll, J. D. (1994). An alternating combinatorial optimization approach to fitting the INDCLUS and generalized INDCLUS models. *Journal of Classification*, *11*, 155–170.

Cliff, N. (1966). Orthogonal rotation to congruence. *Psychometrika*, *31*, 33–42.

Cliff, N. (1973). Scaling. *Annual Review of Psychology*, *24*, 473–506.

Cohen, H. S., & Davison, M. L. (1973). Jiffy-scale: A FORTRAN IV program for generating Ross-ordered pair comparisons. *Behavioral Science*, *18*, 76.

Cohen, H. S., & Jones, L. E. (1973). The effects of random error and subsampling of dimensions on recovery of configurations by non-metric multidimensional scaling. *Psychometrika*, *39*, 69–90.

Commandeur, J. J. F. (1991). *Matching configurations.* Leiden, The Netherlands: DSWO.

Commandeur, J. J. F. (1993). *Missing data in the distance approach to principal component analysis* (Tech. Rep. No. RR-92-07). Leiden, The Netherlands: Department of Data Theory, Leiden University.

Commandeur, J. J. F., & Heiser, W. J. (1993). *Mathematical derivations in the proximity scaling (PROXSCAL) of symmetric data matrices* (Tech. Rep. No. RR-93-03). Leiden, The Netherlands: Department of Data Theory, Leiden University.

Constantine, A. G., & Gower, J. C. (1978). Graphic representations of asymmetric matrices. *Applied Statistics*, *27*, 297–304.

Coombs, C. H. (1950). Psychological scaling without a unit of measurement. *Psychological Review*, *57*, 145–158.

Coombs, C. H. (1964). *A theory of data.* New York: Wiley.

Coombs, C. H. (1967). Thurstone's measurement of social values revisited forty years later. *Journal of Personality and Social Psychology*, *6*, 85–91.

Coombs, C. H. (1975). A note on the relation between the vector model and the unfolding model for preferences. *Psychometrika*, *40*, 115–116.

Coombs, C. H., & Avrunin, G. S. (1977). Single-peaked functions and the theory of preference. *Psychological Review*, *84*, 216–230.

Coombs, C. H., Dawes, R. M., & Tversky, A. (1970). *Mathematical psychology: An elementary introduction.* Englewood-Cliffs, NJ: Prentice-Hall. (Reprinted by Mathesis, Ann Arbor, MI, 1981)

Cooper, L. G. (1972). A new solution to the additive constant problem in metric multidimensional scaling. *Psychometrika*, *37*, 311–322.

Cox, M. A. A., & Cox, T. F. (1992). Interpretation of Stress in non-metric multidimensional scaling. *Statistica Applicata*, *4*, 611–618.

Cox, T. F., & Cox, M. A. A. (1990). Interpreting Stress in multidimensional scaling. *Journal of Statistical Computation and Simulation*, *37*, 211–223.

Cox, T. F., & Cox, M. A. A. (1991). Multidimensional scaling on a sphere. *Communications in Statistics A, Theory and Methods*, *20*, 2943–2953.

Cox, T. F., & Cox, M. A. A. (1994). *Multidimensional scaling.* London: Chapman & Hall.

Coxon, A. P. M., & Jones, C. L. (1978). *The images of occupational prestige: A study in social cognition.* London: Macmillan.

Critchley, F. (1986). Dimensionality theorems in multidimensional scaling and hierarchical cluster analysis. In E. Diday, Y. Escoufier, L. Lebart, J. Lepage, Y. Schektman, & R. Tomassone (Eds.), *Informatics* (Vol. IV, pp. 45–70). Amsterdam: North-Holland.

Critchley, F., & Fichet, B. (1994). The partial order by inclusion of the principal classes of dissimilarity on a finite set and some of their basic properties. In B. V. Cutsem (Ed.), *Classification and dissimilarity analysis* (Vol. 93, pp. 5–65). New York: Springer.

Cross, D. V. (1965a). *Metric properties of multidimensional stimulus control.* Unpublished doctoral dissertation, University of Michigan.

Cross, D. V. (1965b). Metric properties of multidimensional stimulus generalization. In D. I. Mostofsky (Ed.), *Stimulus generalization* (pp. 72–93). San Francisco: Stanford University Press.

Daniels, H. E. (1944). The relation between measures of correlation in the universe of sample permutations. *Biometrika*, *33*, 129–135.

Davison, M. L. (1983). *Multidimensional scaling.* New York: Wiley.

De Boor, C. (1978). *A practical guide to splines.* New York: Springer.

Defays, D. (1978). A short note on a method of seriation. *British Journal of Mathematical and Statistical Psychology*, *31*, 49–53.

De Leeuw, J. (1977). Applications of convex analysis to multidimensional scaling. In J. R. Barra, F. Brodeau, G. Romier, & B. van Cutsem (Eds.), *Recent developments in statistics* (pp. 133–145). Amsterdam, The Netherlands: North-Holland.

De Leeuw, J. (1984). Differentiability of Kruskal's Stress at a local minimum. *Psychometrika*, *49*, 111–113.

De Leeuw, J. (1988). Convergence of the majorization method for multidimensional scaling. *Journal of Classification*, *5*, 163–180.

De Leeuw, J. (1993). *Fitting distances by least squares* (Tech. Rep. No. 130). Los Angeles, CA: Interdivisional Program in Statistics, UCLA.

De Leeuw, J. (1994). Block relaxation algorithms in statistics. In H.-H. Bock, W. Lenski, & M. M. Richter (Eds.), *Information systems and data analysis* (pp. 308–324). Berlin: Springer.

De Leeuw, J., & Groenen, P. J. F. (1997). Inverse multidimensional scaling. *Journal of Classification*, *14*, 3–21.

De Leeuw, J., & Heiser, W. J. (1977). Convergence of correction-matrix algorithms for multidimensional scaling. In J. C. Lingoes, E. E. Roskam, & I. Borg (Eds.), *Geometric representations of relational data* (pp. 735–752). Ann Arbor, MI: Mathesis.

De Leeuw, J., & Heiser, W. J. (1980). Multidimensional scaling with restrictions on the configuration. In P. R. Krishnaiah (Ed.), *Multivariate analysis* (Vol. V, pp. 501–522). Amsterdam, The Netherlands: North-Holland.

De Leeuw, J., & Heiser, W. J. (1982). Theory of multidimensional scaling. In P. R. Krishnaiah & L. N. Kanal (Eds.), *Handbook of statistics* (Vol. 2, pp. 285–316). Amsterdam, The Netherlands: North-Holland.

De Leeuw, J., & Stoop, I. (1984). Upper bounds of Kruskal's Stress. *Psychometrika*, *49*, 391–402.

De Rooij, M., & Heiser, W. J. (2000). Triadic distance models for the analysis of asymmetric three-way proximity data. *British Journal of Mathematical and Statistical Psychology*, *53*, 99–119.

DeSarbo, W. S., Manrai, A. K., & Manrai, L. A. (1994). Latent class multidimensional scaling: A review of recent developments in the marketing and psychometric literature. In R. P. Bagozzi (Ed.), *Advanced methods of marketing research* (pp. 190–222). Cambridge, MA: Blackwell.

DeSarbo, W. S., & Rao, V. R. (1984). GENFOLD2: A set of models and algorithms for the GENeral UnFOLDing analysis of preference/dominance data. *Journal of Classification*, *1*, 147–186.

De Soete, G., Carroll, J. D., & Chaturvedi, A. D. (1993). A modified CANDECOMP method for fitting the extended INDSCAL model. *Journal of Classification*, *10*, 75–92.

De Soete, G., Hubert, L., & Arabie, P. (1988). On the use of simulated annealing for combinatorial data analysis. In W. Gaul & M. Schader (Eds.), *Data, expert knowledge and decisions* (pp. 329–340). Berlin: Springer.

Diederich, G., Messick, S. J., & Tucker, L. R. (1957). A general least squares solution for successive intervals. *Psychometrika*, *22*, 159–173.

Dijksterhuis, G., & Gower, J. C. (1991). The interpretation of generalized Procrustes analysis and allied methods. *Food Quality and Preference*, *3*, 67–87.

Drösler, J. (1979). Foundations of multidimensional metric scaling in Cayley-Klein geometries. *British Journal of Mathematical and Statistical Psychology*, *32*, 185–211.

Drösler, J. (1981). The empirical validity of multidimensional scaling. In I. Borg (Ed.), *Multidimensional data representations: when and why* (pp. 627–651). Ann Arbor, MI: Mathesis.

Dunn-Rankin, P. (1983). *Scaling methods.* Hillsdale, NJ: Lawrence Erlbaum.

Dunn-Rankin, P., Knezek, G. A., Wallace, S., & Zhang, S. (2004). *Scaling methods.* Manwah, NJ: Lawrence Erlbaum.

Eckart, C., & Young, G. (1936). Approximation of one matrix by another of lower rank. *Psychometrika*, *1*, 211–218.

Eisler, H., & Lindman, R. (1990). Representations of dimensional models of similarity. In H. G. Geisler (Ed.), *Psychophysical explorations of mental structures* (pp. 165–171). Toronto: Hogrefe & Huber.

Eisler, H., & Roskam, E. E. (1977). Multidimensional similarity: An experimental and theoretical comparison of vector, distance, and set theoretical models. *Acta Psychologica*, *41*, 1–46 and 335–363.

Ekehammar, B. (1972). Multidimensional scaling according to different vector models for subjective similarity. *Acta Psychologica*, *36*, 79–84.

Ekman, G. (1954). Dimensions of color vision. *Journal of Psychology*, *38*, 467–474.

Ekman, G., Engen, T., Künnapas, T., & Lindman, R. (1964). A quantitative principle of qualitative similarity. *Journal of Experimental Psychology*, *68*, 530–536.

Ekman, G., Goude, G., & Waern, Y. (1961). Subjective similarity in two perceptual continua. *Journal of Experimental Psychology*, *61*, 222–227.

Ekman, G., & Lindman, R. (1961). *Multidimensional ratio scaling and multidimensional similarity* (Reports from the Psychological Laboratories No. 103). University of Stockholm.

Ekman, G. A. (1963). A direct method for multidimensional ratio scaling. *Psychometrika*, *28*, 3–41.

Elizur, D., Borg, I., Hunt, R., & Magyari-Beck, I. (1991). The structure of work values: A cross cultural comparison. *Journal of Organizational Behavior*, *12*, 21–38.

Engen, T., Levy, N., & Schlosberg, H. (1958). The dimensional analysis of a new series of facial expressions. *Journal of Experimental Psychology*, *55*, 454–458.

England, G., & Ruiz-Quintanilla, S. A. (1994). How working is defined: Structure and stability. In I. Borg & S. Dolan (Eds.), *Proceedings of the Fourth International Conference on Work and Organizational Values* (pp. 104–113). Montreal: ISSWOV.

Fechner, G. T. (1860). *Elemente der Psychophysik* (Vol. I and II). Leipzig, Germany: Breitkopf und Hartel.

Feger, H. (1980). Einstellungsstruktur und Einstellungsänderung: Ergebnisse, Probleme und ein Komponentenmodell der Einstellungsobjekte. *Zeitschrift für Sozialpsychologie*, *10*, 331–349.

Fichet, B. (1994). Dimensionality problems in l_1-norm representations. In B. V. Cutsem (Ed.), *Classification and dissimilarity analysis* (Vol. 93, pp. 201–224). New York: Springer.

Fischer, W., & Micko, H. C. (1972). More about metrics of subjective spaces and attention distributions. *Journal of Mathematical Psychology*, *9*, 36–54.

Fletcher, R. (1987). *Practical methods of optimization.* Chichester, England: Wiley.

Foa, U. (1965). New developments in facet design and analysis. *Psychological Review*, *72*, 262–238.

Foa, U. G. (1958). The contiguity principle in the structure of interpersonal relations. *Human Relations*, *11*, 229–238.

Fréchet, M. (1910). Les dimensions d'un ensemble abstrait. *Math. Ann.*, *68*, 145–168.

Gabriel, K. R. (1971). The biplot-graphic display of matrices with applications to principal components analysis. *Biometrika*, *58*, 453–467.

Gaffke, N., & Mathar, R. (1989). A cyclic projection algorithm via duality. *Metrika*, *36*, 29–54.

Galinat, W., & Borg, I. (1987). On symbolic temporal information: beliefs about the experience of duration. *Memory and Cognition*, *15*, 308–317.

Garmize, L. M., & Rychlak, J. F. (1964). Role-play validation of a sociocultural theory of symbolism. *Journal of Consulting Psychology*, *28*, 107–115.

Garner, W. R. (1962). *Uncertainty and structure as psychological concepts.* New York: Wiley.

Garner, W. R. (Ed.). (1974). *The processing of information and structure.* Potomac, MD: Erlbaum.

Gati, I., & Tversky, A. (1982). Representations of qualitative and quantitative dimensions. *Journal of Experimental Psychology: Human Perception and Performance*, *8*, 325–340.

Gifi, A. (1990). *Nonlinear multivariate analysis.* Chichester, England: Wiley.

Gilula, Z., & Haberman, S. J. (1986). Canonical analysis of contingency tables by maximum likelihood. *Journal of the American Statistical Association*, *81*, 780–788.

Gleason, T. C. (1967). *A general model for nonmetric multidimensional scaling.* (University of Michigan, unpublished mimeograph)

Glunt, W., Hayden, T. L., & Liu, W.-M. (1991). The embedding problem for predistance matrices. *Bulletin of Mathematical Biology*, *53*, 769–796.

Glunt, W., Hayden, T. L., & Raydan, M. (1993). Molecular confirmations from distance matrices. *Journal of Computational Chemistry*, *14*, 114–120.

Glushko, R. J. (1975). Pattern goodness and redundancy revisited: Multidimensional scaling and hierarchical clustering analyses. *Perception & Psychophysics*, *17*, 158–162.

Goodman, L. A. (1985). The analysis of cross-classified data having ordered and/or unordered categories: Association models, correlation models and asymmetry models for contingency tables with or without missing values. *The Annals of Statistics*, *13*, 10–69.

Goodman, L. A. (1986). Some useful extensions to the usual correspondence analysis approach and the usual loglinear approach in the analysis of contingency tables (with comments). *International Statistical Review*, *54*, 243–309.

Goude, G. (1972). A multidimensional scaling approach to the perception of art. *Scandinavian Journal of Psychology*, *13*, 258–271 and 272–284.

Gower, J. C. (1966). Some distance properties of latent root and vector methods used in multivariate analysis. *Biometrika*, *53*, 325–338.

Gower, J. C. (1971). A general coefficient of similarity and some of its properties. *Biometrics*, *27*, 857–874.

Gower, J. C. (1975). Generalized Procrustes analysis. *Psychometrika*, *40*, 33–51.

Gower, J. C. (1977). The analysis of asymmetry and orthogonality. In F. Brodeau, G. Romier, & B. Van Cutsem (Eds.), *Recent developments in statistics* (pp. 109–123). Amsterdam: North-Holland.

Gower, J. C. (1985). Measures of similarity, dissimilarity, and distance. In S. Kotz, N. L. Johnson, & C. B. Read (Eds.), *Encyclopedia of statistical sciences* (Vol. 5, pp. 397–405). New York: Wiley.

Gower, J. C., & Dijksterhuis, G. B. (2004). *Procrustes problems.* Oxford: Oxford University Press.

Gower, J. C., & Groenen, P. J. F. (1991). Applications of the modified Leverrier-Faddeev algorithm for the construction of explicit matrix spectral decompositions and inverses. *Utilitas Mathematica*, *40*, 51–64.

Gower, J. C., & Hand, D. J. (1996). *Biplots.* London: Chapman & Hall.

Gower, J. C., & Legendre, P. (1986). Metric and Euclidean properties of dissimilarity coefficients. *Journal of Classification*, *3*, 5–48.

Graef, J., & Spence, I. (1979). Using distance information in the design of large multidimensional scaling experiments. *Psychological Bulletin*, *86*, 60–66.

Gramlich, G. (2004). *Anwendungen der linearen Algebra.* München, Germany: Fachbuchverlag Leipzig.

Green, B. F. (1952). The orthogonal approximation of an oblique structure in factor analysis. *Psychometrika*, *17*, 429–440.

Green, P. E. (1974). *On the robustness of multidimensional scaling techniques.* Unpublished paper. University of Pennsylvania, Wharton Business School.

Green, P. E., & Carmone, F. J. (1970). *Multidimensional scaling and related techniques in marketing analysis.* Boston, MA: Allyn & Bacon.

Green, P. E., & Carroll, R. D. (1976). *Mathematical tools for applied multivariate analysis.* New York: Academic.

Green, P. E., & Rao, V. (1972). *Applied multidimensional scaling.* Hinsdale, IL: Dryden.

Green, P. E., & Wind, Y. (1973). *Multivariate decisions in marketing: A measurement approach.* Hinsdale, IL: Dryden.

Greenacre, M., & Blasius, J. (Eds.). (1994). *Correspondence analysis in the social sciences.* London: Academic.

Greenacre, M. J. (1984). *Theory and applications of correspondence analysis.* New York: Academic.

Greenacre, M. J. (1994). *Correspondence analysis in practice.* London: Academic.

Greenberg, M. J. (1980). *Euclidean and non-Euclidean geometries.* San Francisco: Freeman.

Groenen, P. J. F. (1993). *The majorization approach to multidimensional scaling: Some problems and extensions.* Leiden, The Netherlands: DSWO.

Groenen, P. J. F., De Leeuw, J., & Mathar, R. (1996). Least squares multidimensional scaling with transformed distances. In W. Gaul & D. Pfeifer (Eds.), *Studies in classification, data analysis, and knowledge organization* (pp. 177–185). Berlin: Springer.

Groenen, P. J. F., & Franses, P. H. (2000). Visualizing time-varying correlations across stock markets. *Journal of Empirical Finance*, *7*, 155–172.

Groenen, P. J. F., & Gifi, A. (1989). *Anacor* (Tech. Rep. No. UG-89-01). Leiden, The Netherlands: Department of Data Theory, Leiden University.

Groenen, P. J. F., & Heiser, W. J. (1991). *An improved tunneling function for finding a decreasing series of local minima* (Tech. Rep. No. RR-91-06). Leiden, The Netherlands: Department of Data Theory, Leiden University.

Groenen, P. J. F., & Heiser, W. J. (1996). The tunneling method for global optimization in multidimensional scaling. *Psychometrika*, *61*, 529–550.

Groenen, P. J. F., & Heiser, W. J. (2000). Iterative majorization algorithms in statistical computing. *Journal of Computational and Graphical Statistics*, *9*, 44–48.

Groenen, P. J. F., Heiser, W. J., & Meulman, J. J. (1999). Global optimization in least-squares multidimensional scaling by distance smoothing. *Journal of Classification*, *16*, 225–254.

Groenen, P. J. F., Mathar, R., & Heiser, W. J. (1995). The majorization approach to multidimensional scaling for Minkowski distances. *Journal of Classification*, *12*, 3–19.

Groenen, P. J. F., Mathar, R., & Trejos, J. (2000). Global optimization methods for multidimensional scaling applied to mobile communication. In W. Gaul, O. Opitz, & M. Schader (Eds.), *Data analysis* (p. 459-469). Heidelberg: Springer.

Groenen, P. J. F., & Meulman, J. J. (2004). A comparison of the ratio of variances in distance-based and classicial multivariate analysis. *Statistica Neerlandica*, *58*, 428–439.

Groenen, P. J. F., & Van de Velden, M. (2004). Inverse correspondence analysis. *Linear Algebra and its Applications*, *388*, 221–238.

Gulliksen, H. (1946). Paired comparisons and the logic of measurement. *Psychological Review*, *53*, 199–213.

Guthrie, J. T. (1973). Models of reading and reading disability. *Journal of Educational Psychology*, *65*, 9–18.

Guttman, L. (1941). The quantification of a class of attributes: A theory and method of scale construction. In P. Horst et al. (Eds.), *The prediction of personal adjustment* (pp. 319–348). New York: Social Science Research Council.

Guttman, L. (1954). A new approach to factor analysis: The radex. In P. Lazarsfeld (Ed.), *Mathematical thinking in the behavioral sciences* (pp. 258–348). New York: Free Press.

Guttman, L. (1959). A structural theory for intergroup beliefs and attitude. *American Sociological Review*, *24*, 318–328.

Guttman, L. (1965). A faceted definition of intelligence. *Scripta Hierosolymitana*, *14*, 166–181.

Guttman, L. (1968). A general nonmetric technique for finding the smallest coordinate space for a configuration of points. *Psychometrika*, *33*, 469–506.

Guttman, L. (1971). Measurement as structural theory. *Psychometrika*, *36*, 329–347.

Guttman, L. (1976). *Faceted SSA* (Tech. Rep.). Jerusalem, Israel: The Israel Institute of Applied Social Research.

Guttman, L. (1977). What is not what in statistics. *The Statistician*, *26*, 81–107.

Guttman, L. (1981). Efficacy coefficients for differences among averages. In I. Borg (Ed.), *Multidimensional data representations: when and why* (pp. 1–10). Ann Arbor, MI: Mathesis.

Guttman, L. (1982). Facet theory, smallest space analysis, and factor analysis. *Perceptual and Motor Skills*, *54*, 491–493.

Guttman, L. (1985). Coefficients of polytonicity and monotonicity. In S. Kotz & N. L. Johnson (Eds.), *Encyclopedia of statistical sciences* (Vol. 7, pp. 80–87). New York: Wiley.

Guttman, L. (1991). *Louis Guttman: In Memoriam - Chapters from an unfinished textbook on facet theory.* Jerusalem: The Israel Academy of Sciences and Humanities.

Guttman, L., & Levy, S. (1991). Two structural laws for intelligence tests. *Intelligence*, *15*, 79–103.

Guttman, N., & Kalish, H. I. (1956). Discriminability and stimulus generalization. *Journal of Experimental Psychology*, *51*, 79–88.

Harshman, R. A. (1972). Determination and proof of minimum uniqueness conditions for PARAFAC-I. *UCLA Working Papers in Phonetics*, *22*.

Harshman, R. A., & Lundy, M. E. (1984). The Parafac model for three-way factor analysis and multidimensional scaling. In H. G. Law, C. W. Snyder, J. A. Hattie, & R. P. McDonald (Eds.), *Research methods for multimode data analysis* (pp. 122–215). New York: Praeger.

Hefner, R. A. (1958). *Extension of the law of comparative judgment to discriminable and multidimensional stimuli.* Unpublished doctoral dissertation, University of Michigan.

Heiser, W. J. (1981). *Unfolding analysis of proximity data.* Unpublished doctoral dissertation, Leiden University.

Heiser, W. J. (1985). *Multidimensional scaling by optimizing goodness-of-fit to a smooth hypothesis* (Tech. Rep. No. RR-85-07). Leiden, The Netherlands: Department of Data Theory, Leiden University.

Heiser, W. J. (1988a). Multidimensional scaling with least absolute residuals. In H. H. Bock (Ed.), *Classification and related methods* (pp. 455–462). Amsterdam: North-Holland.

Heiser, W. J. (1988b). PROXSCAL, multidimensional scaling of proximities. In A. Di Ciaccio & G. Bove (Eds.), *International meeting on the analysis of multiway data matrices, software guide* (pp. 77–81). Rome: C.N.R.

Heiser, W. J. (1989a). Order invariant unfolding analysis under smoothness restrictions. In G. De Soete, H. Feger, & K. C. Klauer (Eds.), *New developments in psychological choice modeling* (pp. 3–31). Amsterdam: Elsevier Science.

Heiser, W. J. (1989b). The city-block model for three-way multidimensional scaling. In R. Coppi & S. Bolasco (Eds.), *Multiway data analysis* (pp. 395–404). Amsterdam: Elsevier Science.

Heiser, W. J. (1990). A generalized majorization method for least squares multidimensional scaling of pseudodistances that may be negative. *Psychometrika*, *56*, 7–27.

Heiser, W. J. (1993). Clustering in low-dimensional space. In O. Opitz, B. Lausen, & R. Klar (Eds.), *Information and classification: Concepts, methods, and applications* (pp. 162–173). Berlin: Springer.

Heiser, W. J. (1995). Convergent computation by iterative majorization: Theory and applications in multidimensional data analysis. In W. J. Krzanowski (Ed.), *Recent advances in descriptive multivariate analysis* (pp. 157–189). Oxford: Oxford University Press.

Heiser, W. J., & De Leeuw, J. (1979). *How to use SMACOF-III: A program for metric multidimensional unfolding* (Tech. Rep.). Leiden University, Department of Data Theory.

Heiser, W. J., & Groenen, P. J. F. (1997). Cluster differences scaling with a within-cluster loss component and a fuzzy successive approximation strategy to avoid local minima. *Psychometrika*, *62*, 529–550.

Heiser, W. J., & Meulman, J. J. (1983a). Analyzing rectangular tables by joint and constrained MDS. *Journal of Econometrics*, *22*, 193–167.

Heiser, W. J., & Meulman, J. J. (1983b). Constrained multidimensional scaling including confirmation. *Applied Psychological Measurement*, *7*, 381–404.

Helm, C. E. (1959). *A multidimensional ratio scaling analysis of color relations* (Tech. Rep.). Princeton, NJ: Princeton University and Educational Testing Service.

Helm, C. E. (1964). Multidimensional ratio scaling analysis of perceived color relations. *Journal of the Optical Society of America*, *54*, 256–262.

Hirschfeld, H. O. (1935). A connection between correlation and contingency. *Proceedings Cambridge Philosophical Society*, *31*, 520–524.

Horan, C. B. (1969). Multidimensional scaling: Combining observations when individuals have different perceptual structures. *Psychometrika*, *34*, 139–165.

Hotelling, H. (1933). Analysis of a complex of statistical variables into principal components. *Journal of Educational Psychology*, *24*, 417–441, 498–520.

Huber, P. J. (1964). Robust estimation of a location parameter. *Annals of Mathematical Statistics*, *35*, 73–101.

Hubert, L., Arabie, P., & Hesson-McInnis, M. (1992). Multidimensional scaling in the city-block metric: A combinatorial approach. *Journal of Classification*, *9*, 211–236.

Hubert, L. J., & Arabie, P. (1986). Unidimensional scaling and combinatorial optimization. In J. De Leeuw, W. J. Heiser, J. J. Meulman, & F. Critchley (Eds.), *Multidimensional data analysis* (pp. 181–196). Leiden, The Netherlands: DSWO.

Hubert, L. J., Arabie, P., & Meulman, J. J. (1997). Linear and circular unidimensional scaling for symmetric proximity matrices. *British Journal of Mathematical and Statistical Psychology*, *50*, 253–284.

Hubert, L. J., & Golledge, R. G. (1981). Matrix reorganisation and dynamic programming: Applications to paired comparison and unidimensional seriation. *Psychometrika*, *46*, 429–441.

Hunter, D. R., & Lange, K. (2004). A autorial on MM algorithms. *The American Statistician*, *39*, 30–37.

Hurley, J. R., & Cattell, R. B. (1962). The Procrustes program: Producing direct rotation to test a hypothesized factor structure. *Behavioral Science*, *7*, 258–262.

Indow, T. (1974). Applications of multidimensional scaling in perception. In E. C. Carterette & M. P. Friedman (Eds.), *Handbook of perception* (Vol. II, pp. 493–531). New York: Academic.

Isaac, P. D., & Poor, D. (1974). On the determination of appropriate dimensionality in data with error. *Psychometrika*, *39*, 91–109.

Kaiser, H. F. (1958). The varimax criterion for analytic rotation in factor analysis. *Psychometrika*, *23*, 187–200.

Kearsley, A. J., Tapia, R. A., & Trosset, M. W. (1998). The solution of the metric Stress and S-Stress problems in multidimensional scaling using Newton's method. *Computational Statistics*, *13*, 369–396.

Kendall, D. G. (1971). Seriation from abundance matrices. In F. R. Hodson, D. G. Kendall, & P. Tautu (Eds.), *Mathematics in the archaelogical and historical sciences.* Edinburgh: Edinburgh University Press.

Kiers, H. A. L. (1990). Majorization as a tool for optimizing a class of matrix functions. *Psychometrika*, *55*, 417–428.

Kiers, H. A. L. (2002). Setting up alternating least squares and iterative majorization algorithms for solving various matrix optimization problems. *Computational Statistics and Data Analysis*, *41*, 157–170.

Kiers, H. A. L., & Groenen, P. J. F. (1996). A monotonicaly convergent algorithm for orthogonal congruence rotation. *Psychometrika*, *61*, 375–389.

Kim, C., Rangaswamy, A., & DeSarbo, W. S. (1999). A quasi-metric approach to multidimensional unfolding for reducing the occurrence of degenerate solutions. *Multivariate Behavioral Research*, *34*, 143–180.

Klahr, D. (1969). A Monte-Carlo investigation of the statistical significance of Kruskal's nonmetric scaling procedure. *Psychometrika*, *34*, 319–330.

Kloek, T., & Theil, H. (1965). International comparisons of prices and quantities consumed. *Econometrica*, *33*, 535–556.

Krantz, D. H. (1967). Rational distance functions for multidimensional scaling. *Journal of Mathematical Psychology*, *4*, 226–245.

Krantz, D. H. (1972). Integration of just noticeable differences. *Journal of Mathematical Psychology*, *8*, 591–599.

Krantz, D. H., Luce, R. D., Suppes, P., & Tversky, A. (1971). *Foundations of measurement* (Vol. 1). New York: Academic.

Krantz, D. H., & Tversky, A. (1975). Similarity of rectangles: An analysis of subjective dimensions. *Journal of Mathematical Psychology*, *12*, 4–34.

Kristof, W. (1970). A theorem on the trace of certain matrix products and some applications. *Journal of Mathematical Psychology*, *7*, 515–530.

Kristof, W., & Wingersky, B. (1971). Generalization of the orthogonal Procrustes rotation procedure for more than two matrices. In *Proceedings of the 79th annual convention of the american psychological association* (pp. 89–90).

Krumhansl, C. L. (1978). Concerning the applicability of geometric models to similarity data: The interrelationship between similarity and spatial density. *Psychological Review*, *85*, 445–463.

Kruskal, J. B. (1964a). Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. *Psychometrika*, *29*, 1–27.

Kruskal, J. B. (1964b). Nonmetric multidimensional scaling: A numerical method. *Psychometrika*, *29*, 115–129.

Kruskal, J. B. (1968). *How to use M-D-SCAL, a program to do multidimensional scaling and multidimensional unfolding (Version 4 and 4M Fortran IV)* (Tech. Rep.). Murray Hill, NJ: Bell Telephone Laboratories.

Kruskal, J. B. (1977). Multidimensional scaling and other methods for discovering structure. In K. Enslein, A. Ralston, & H. S. Wilf (Eds.), *Statistical methods for digitial computers* (Vol. 2, pp. 296–339). New York: Wiley.

Kruskal, J. B., & Carmone, F. J. (1969). *How to use M-D-SCAL Version 5M and other useful information* (Tech. Rep.). Murray Hill, NJ: Bell Labs.

Kruskal, J. B., & Carroll, J. D. (1969). Geometrical models and badness-of-fit functions. In P. R. Krishnaiah (Ed.), *Multivariate analysis* (Vol. 2, pp. 639–671). New York: Academic.

Kruskal, J. B., & Wish, M. (1978). *Multidimensional scaling.* Beverly Hills, CA: Sage.

Kruskal, J. B., Young, F. W., & Seery, J. B. (1978). *How to use KYST-2, a very flexible program to do multidimensional scaling and unfolding* (Tech. Rep.). Murray Hill, NJ: Bell Labs.

Lange, K., Hunter, D. R., & Yang, I. (2000). Optimization transfer using surrogate objective functions. *Journal of Computational and Graphical Statistics*, *9*, 1–20.

Langeheine, R. (1980a). *Approximate norms and significance tests for the Lingoes-Borg Procrustean Individual Differences Scaling PINDIS* (Tech. Rep. No. 39). Kiel: Institut für die Pädagogik der Naturwissenschaften, Universität Kiel.

Langeheine, R. (1980b). Erwartete Fitwerte für Zufallskonfigurationen in PINDIS. *Zeitschrift für Sozialpsychologie*, *11*, 38–49.

Langeheine, R. (1982). Statistical evaluation of measures of fit in the Lingoes-Borg Procrustean individual differences scaling. *Psychometrika*, *47*, 427–442.

Lawler, E. (1967). The multitrait-multimethod approach to measuring managerial job performance. *Journal of Applied Psychology*, *51*, 369–381.

Lawson, C. L., & Hanson, R. J. (1974). *Solving least squares problems.* Englewood Cliffs, NJ: Prentice-Hall.

Lazarte, A., & Schönemann, P. H. (1991). Saliency metric for subadditive dissimilarity judgments of rectangles. *Perception and Psychophysics*, *49*, 142–158.

Lebart, L., Morineau, A., & Warwick, K. M. (1984). *Multivariate descriptive statistical analysis.* New York: Wiley.

Leutner, D., & Borg, I. (1983). Zufallskritische Beurteilung der Übereinstimmung von Faktor- und MDS-Konfigurationen. *Diagnostica*, *24*, 320–335.

Levelt, W. J. M., Geer, J. P. Van de, & Plomp, R. (1966). Triadic comparisons of musical intervals. *British Journal of Mathematical and Statistical Psychology*, *19*, 163–179.

Levy, S. (1976). Use of the mapping sentence for coordinating theory and research: A cross-cultural example. *Quality and Quantity*, *10*, 117–125.

Levy, S. (1983). A cross-cultural analysis of the structure and levels of atttitudes towards acts of political protest. *Social Indicators Research*, *12*, 281–309.

Levy, S., & Guttman, L. (1975). On the multivariate structure of wellbeing. *Social Indicators Research*, *2*, 361–388.

Lindman, H., & Caelli, T. (1978). Constant curvature Riemannean scaling. *Journal of Mathematical Psychology*, *17*, 89–109.

Lingoes, J. C. (1965). An IBM-7090 program for Guttman-Lingoes smallest space analysis-I. *Behavioral Science*, *10*, 183–184.

Lingoes, J. C. (1971). Some boundary conditions for a monotone analysis of symmetric matrices. *Psychometrika*, *36*, 195–203.

Lingoes, J. C. (1973). *The Guttman-Lingoes nonmetric program series.* Ann Arbor, MI: Mathesis.

Lingoes, J. C. (1981). Testing regional hypotheses in multidimensional scaling. In I. Borg (Ed.), *Multidimensional data representations: when and why* (pp. 280–310). Ann Arbor, MI: Mathesis.

Lingoes, J. C. (1989). *Guttman-Lingoes nonmetric PC series manual.* Ann Arbor, MI: Mathesis.

Lingoes, J. C., & Borg, I. (1977). Identifying spatial manifolds for interpretation. In E. E. Roskam, J. C. Lingoes, & I. Borg (Eds.), *Geometric representations of relational data.* Ann Arbor, MI: Mathesis.

Lingoes, J. C., & Borg, I. (1978). A direct approach to individual differences scaling using increasingly complex transformations. *Psychometrika*, *43*, 491–519.

Lingoes, J. C., & Borg, I. (1983). A quasi-statistical model for choosing between alternative configurations derived from ordinally constrained data. *British Journal of Mathematical and Statistical Psychology*, *36*, 36–53.

Lingoes, J. C., & Guttman, L. (1967). Nonmetric factor analysis: A rank reducing alternative to linear factor analysis. *Multivariate Behavioral Research*, *2*, 485–505.

Lingoes, J. C., & Roskam, E. E. (1973). A mathematical and empirical analysis of two multidimensional scaling algorithms. *Psychometrika*, *38*.

Luce, R. D. (1959). *Individual choice behavior.* New York: Wiley.

Luce, R. D. (1961). A choice theory analysis of similarity judgments. *Psychometrika*, *26*, 151–163.

Luce, R. D., & Suppes, P. (1963). Preference, utility, and subjective probability. In R. D. Luce, R. R. Bush, & E. Galanter (Eds.), *Handbook of mathematical psychology* (Vol. 3, pp. 249–410). New York: Wiley.

Luengo, F., Raydan, M., Glunt, W., & Hayden, T. L. (2002). Preconditioned spectral gradient method. *Numerical Algorithms*, *30*, 241-258.

Lüer, G., & Fillbrandt, H. (1970). Interindividuelle Unterschiede bei der Beurteilung von Reizähnlichkeiten. *Zeitschrift für experimentelle und angewandte Psychologie*, *18*, 123–138.

Lüer, G., Osterloh, W., & Ruge, A. (1970). Versuche zur multidimensionalen Skalierung von subjektiven Reizähnlichkeiten auf Grund von Schätzurteilen und Verwechselungshäufigkeiten beim Wiedererkennen. *Psychologische Forschung*, *33*, 223–241.

MacCallum, R. C. (1976). Effects on INDSCAL of non-orthogonal perceptions of object space dimensions. *Psychometrika*, *41*, 177–188.

MacCallum, R. C. (1977). Effects of conditionality on INDSCAL and ALSCAL weights. *Psychometrika*, *42*, 297–305.

Mathar, R. (1989). Algorithms in multidimensional sclaing. In O. Opitz (Ed.), *Conceptual and numerical analysis of data* (pp. 159–177). Berlin: Springer.

Mathar, R. (1994). Multidimensional scaling with l_p-distances: A unifying approach. In H. H. Bock, W. Lenski, & M. M. Richter (Eds.), *Information systems and data analysis* (pp. 325–331). Heidelberg, Germany: Springer.

Mathar, R. (1997). *Multidimensionale Skalierung.* Stuttgart: B. G. Teubner.

Mathar, R., & Groenen, P. J. F. (1991). Algorithms in convex analysis applied to multidimensional scaling. In E. Diday & Y. Lechevallier (Eds.), *Symbolic-numeric data analysis and learning* (pp. 45–56). Commack, NY: Nova Science.

Mathar, R., & Meyer, R. (1993). Preorderings, monotone functions, and best rank r approximations with applications to classical MDS. *Journal of Statistical Planning and Inference*, *37*, 291–305.

Mathar, R., & Žilinskas, A. (1993). On global optimization in two dimensional scaling. *Acta Aplicandae Mathematica*, *33*, 109–118.

McGee, V. E. (1966). The multidimensional scaling of "elastic" distances. *British Journal of Mathematical and Statistical Psychology*, *19*, 181–196.

Merkle, E. (1981). *Die Erfassung und Nutzung von Informationen über den Sortimentsverbund in Handelsbetrieben.* Berlin: Duncker & Humbolt.

Messick, S. J., & Abelson, R. P. (1956). The additive constant problem in multidimensional scaling. *Psychometrika*, *21*, 1–15.

Meulman, J. J. (1986). *A distance approach to nonlinear multivariate analysis.* Leiden, The Netherlands: DSWO.

Meulman, J. J. (1992). The integration of multidimensional scaling and multivariate analysis with optimal transformations. *Psychometrika*, *57*, 539–565.

Meulman, J. J., Heiser, W. J., & De Leeuw, J. (1983). *Progress notes on SMACOF-II* (Tech. Rep. No. UG-83). Leiden, The Netherlands: Department of Data Theory, Leiden University.

Meulman, J. J., Heiser, W. J., & SPSS. (1999). *Spss categories 10.0.* Chicago: SPSS.

Meyer, R. (1993). *Matrix-Approximation in der Multivariaten Statistik.* Aachen, Germany: Verlag der Augustinus Buchhandlung.

Mezzich, J. E. (1978). Evaluating clustering methods for psychiatric diagnosis. *Biological Psychiatry*, *13*, 265–281.

Micko, H. C., & Fischer, W. (1970). The metric of multidimensional psychological spaces as a function of the differential attention to subjective attributes. *Journal of Mathematical Psychology*, *7*, 118–143.

Miller, G. A. (1956). The magical number seven, plus minus one: Some limits on our capacity for processing information. *Psychological Review*, *63*, 81–97.

Möbus, C. (1975). Bemerkungen zur Skalierung interindividueller Urteilsdifferenzen: Simulation des INDSCAL-Modells von Carroll & Chang mit der 'Points-of-View' Analyse von Tucker & Messick. *Archiv für Psychologie*, *127*, 189–209.

Möbus, C. (1979). Zur Analyse nichtsymmetrischer Ähnlichkeitsurteile: Ein dimensionales Driftmodell, eine Vergleichshypothese, Tversky's Kontrastmodell und seine Fokushypothese. *Archiv für Psychologie*, *131*, 105–136.

Mooijaart, A., & Commandeur, J. J. F. (1990). A general solution of the weighted orthonormal Procrustes problem. *Psychometrika*, *53*, 657–663.

Moorhead, G., & Griffin, R. W. (1989). *Organizational behavior.* Boston: Houghton Mifflin.

Mulaik, S. A. (1972). *The foundations of factor analysis.* New York: McGraw-Hill.

Müller, C. (1984). *Mehrdimensionale nicht-numerische Skalierung in der Psychologie.* Göttingen, Germany: Hogrefe.

Nishisato, S. (1980). *Dual scaling and its applications* (Vol. 24). Toronto: University of Toronto Press.

Nishisato, S. (1994). *Elements of dual scaling: An introduction to practical data analysis.* Hillsdale, NJ: Erlbaum.

Noma, E., & Johnson, J. (1977). *Constraining nonmetric multidimensional scaling configurations* (Tech. Rep. No. 60). Human Performance Center, University of Michigan.

Norpoth, H. (1979a). Dimensionen des Parteienkonflikts und Präferenzordnungen der deutschen Wählerschaft: Eine Unfoldinganalyse. *Zeitschrift für Sozialpsychologie*, *10*, 350–362.

Norpoth, H. (1979b). The parties come to order! Dimensions of preferential choice in the West German electorate, 1961-1976. *American Political Science Review*, *73*, 724–736.

Okada, A. (1990). A generalization of asymmetric multidimensional scaling. In M. Schader & W. Gaul (Eds.), *Knowledge, data and computer-assited decisions* (pp. 127–138). Berlin: Springer.

Okada, A., & Imaizumi, T. (1987). Geometric models for asymmetric similarity data. *Behaviormetrika*, *21*, 81–96.

Okada, A., & Imaizumi, T. (1997). Asymmetric multidimensional scaling of two-mode three-way proximities. *Journal of Classification*, *14*, 195–224.

Ortega, J. M., & Rheinboldt, W. C. (1970). *Iterative solutions of nonlinear equations in several variables.* New York: Academic.

Pearson, K. (1896). Regression, heridity, and panmixia. *Philosophical Transactions of the Royal Society of London, Ser. A*, *187*, 269–318.

Pearson, K. (1901). On lines and planes of closest fit to a system of points in space. *Philosophical Magazine*, *2*, 557–572.

Pease, I., M. C. (1965). *Methods of matrix algebra.* New York: Academic.

Peay, E. R. (1988). Multidimensional rotation and scaling of configurations to optimal agreement. *Psychometrika*, *53*, 199–208.

Pliner, V. (1996). Metric, unidimensional scaling and global optimization. *Journal of Classification*, *13*, 3–18.

Poole, K. T. (1984). Least squares metric, unidimensional unfolding. *Psychometrika*, *49*, 311–323.

Poole, K. T. (1990). Least squares metric, unidimensional scaling of multivariate linear models. *Psychometrika*, *55*, 123–149.

Poor, D., & Wherry, R. J. (1976). Invariance of multidimensional configurations. *British Journal of Mathematical and Statistical Psychology*, *26*, 114–125.

Porrat, R. (1974). *A laboratory manual for the Guttman-Lingoes nonmetric computer programs* (Tech. Rep. No. RP 456 E). The Israel Institute of Applied Social Research.

Rabinowitz, G. (1976). A procedure for ordering object pairs consistent with the multidimensional unfolding model. *Psychometrika*, *41*, 349–373.

Rabinowitz, G. B. (1975). An introduction to nonmetric multidimensional scaling. *American Journal of Political Science*, *19*, 343–390.

Ramsay, J. O. (1969). Some statistical considerations in multidimensional scaling. *Psychometrika*, *34*, 167–182.

Ramsay, J. O. (1977). Maximum likelihood estimation in multidimensional scaling. *Psychometrika*, *42*, 241–266.

Ramsay, J. O. (1988). Monotone regression splines in action. *Statistical Science*, *3*(4), 425–461.

Restle, F. (1959). A metric and an ordering on sets. *Psychometrika*, *24*, 207–220.

Richardson, M., & Kuder, G. F. (1933). Making a rating scale that measures. *Personnel Journal*, *12*, 36–40.

Richardson, M. W. (1938). Multidimensional psychophysics. *Psychological Bulletin*, *35*, 659–660.

Rosenberg, S., & Kim, M. P. (1975). The method of sorting as a data gathering procedure in multivariate research. *Multivariate Behavioral Research*, *10*, 489–502.

Roskam, E. E. (1972). *An algorithm for multidimensional scaling by metric transformations of data* (Program Bulletin No. 23 106). Nijmegen, The Netherlands: Universiteit Nijmegen.

Roskam, E. E. (1979a). A general system for nonmetric data analysis. In J. C. Lingoes, E. E. Roskam, & I. Borg (Eds.), *Geometric representations of relational data* (pp. 313–347). Ann Arbor, MI: Mathesis.

Roskam, E. E. (1979b). The nature of data: Interpretation and representation. In J. C. Lingoes, E. E. Roskam, & I. Borg (Eds.), *Geometric representations of relational data* (pp. 149–235). Ann Arbor, MI: Mathesis.

Roskam, E. E., & Lingoes, J. C. (1981). Minissa. In S. S. Schiffman, M. L. Reynolds, & F. W.Young (Eds.), *Introduction to multidimensional scaling* (pp. 362–371). New York: Academic.

Ross, R. T. (1934). Optimum orders for presentations of pairs in paired comparisons. *Journal of Educational Psychology*, *25*, 375–382.

Rothkopf, E. Z. (1957). A measure of stimulus similarity and errors in some paired-associate learning. *Journal of Experimental Psychology*, *53*, 94–101.

Sammon, J. W. (1969). A non-linear mapping for data structure analysis. *IEEE Transactions on Computers*, *18*, 401–409.

SAS. (1999). Electronic helpfiles, retrieved October 22, 2004, from `http://rocs.acomp.usf.edu/sas/sashtml/stat/chap53/sect25.htm`

Schönemann, P. H. (1966). A generalized solution of the orthogonal Procrustes problem. *Psychometrika*, *31*, 1–10.

Schönemann, P. H. (1970). On metric multidimensional unfolding. *Psychometrika*, *35*, 349–366.

Schönemann, P. H. (1972). An algebraic solution for a class of subjective metrics models. *Psychometrika*, *37*, 441–451.

Schönemann, P. H. (1982). A metric for bounded response scales. *Bulletin of the Psychonomic Society*, *19*, 317–319.

Schönemann, P. H. (1983). Some theory and results for metrics for bounded response scales. *Journal of Mathematical Psychology*, *27*, 311–324.

Schönemann, P. H. (1985). On the formal differentiation of traces and determinants. *Journal of Multivariate Behavioral Research*, *20*, 113–139.

Schönemann, P. H. (1990). Psychophysical maps for rectangles. In H. G. Geissler (Ed.), *Psychophysical explorations of mental structures* (pp. 149–164). Toronto: Hogrefe & Huber.

Schönemann, P. H. (1994). Measurement: The reasonable ineffectiveness of mathematics in the social sciences. In I. Borg & P. P. Mohler (Eds.), *Trends and perspectives in empirical social research* (pp. 149–160). New York: De Gruyter.

Schönemann, P. H., & Borg, I. (1981a). Measurement, scaling, and factor analysis. In I. Borg (Ed.), *Multidimensional data representations: When and why* (pp. 380–419). Ann Arbor, MI: Mathesis.

Schönemann, P. H., & Borg, I. (1981b). On the interaction between area and shape. In I. Borg (Ed.), *Multidimensional data representations: When and why* (pp. 432–440). Ann Arbor, MI: Mathesis.

Schönemann, P. H., & Borg, I. (1983). Grundlagen der metrischen mehrdimensionalen Skaliermethoden. In H. Feger & J. Bredenkamp (Eds.), *Enzyklopädie der Psychologie: Messen und Testen* (pp. 257–345). Göttingen: Hofgrefe.

Schönemann, P. H., & Carroll, R. M. (1970). Fitting one matrix to another under choice of a central dilation and a rigid motion. *Psychometrika*, *35*, 245–256.

Schönemann, P. H., Dorcey, T., & Kienapple, K. (1985). Subadditive concatenation in dissimilarity judgments. *Perception and Psychophysics*, *38*, 1–17.

Schönemann, P. H., James, W. L., & Carter, F. S. (1978). COSPA, Common Space Analysis - a program for fitting and testing Horan's subjective metrics model. *Journal of Marketing Research*, *15*, 268–270.

Schönemann, P. H., James, W. L., & Carter, F. S. (1979). Statistical inference in multidimensional scaling: A method for fitting and testing Horan's Model. In J. C. Lingoes, E. E. Roskam, & I. Borg (Eds.), *Geometric representations of relational data* (pp. 791–826). Ann Arbor, MI: Mathesis.

Schönemann, P. H., & Lazarte, A. (1987). Psychophysical maps for subadditive dissimilarity ratings. *Perception and Psychophysics*, *42*, 342–354.

Schönemann, P. H., & Wang, M. M. (1972). An individual differences model for multidimensional scaling of preference data. *Psychometrika*, *37*, 275–311.

Schulz, U. (1972). Ein Euklidisches Modell der multidimensionalen Skalierung unter Berücksichtigung individueller Differenzen. In L. Eckensberger (Ed.), *Bericht über den 28. Kongress der Deutschen Gesellschaft für Psychologie* (pp. 75–89). Saarbrücken, Germany.

Schulz, U. (1975). Zu einem Dekompositionsmodell der multidimensionalen Skalierung mit individueller Gewichtung der Dimensionen. *Psychologische Beiträge*, *17*, 167–187.

Schulz, U. (1980). An alternative procedure for the analysis of similarity data and its comparison to the Idioscal- and Indscal- procedure. In H. Feger & E. D. Lantermann (Eds.), *Similarity and choice* (pp. 140–149). Bern, Switzerland: Huber.

Schulz, U., & Pittner, P. M. (1978). Zur multidimensionalen Skalierung individueller Differenzen. *Psychologische Beiträge*, *20*, 294–315.

Schumaker, L. (1981). *Spline functions: Basic theory.* New York: Wiley.

Schwarz, S., & Bilsky, W. (1987). Toward a universal psychological structure of human values. *Journal of Personality and Social Psychology*, *53*, 550–562.

Searle, S. R. (1982). *Matrix algebra useful for statistics.* New York: Wiley.

Shepard, R. N. (1957). Stimulus and response generalization: A stochastic model for relating generalization to distance in psychological space. *Psychometrika*, *22*, 325–345.

Shepard, R. N. (1958a). Stimulus and response generalization: Deduction of the generalization gradient from a trace model. *Psychological Review*, *65*, 242–256.

Shepard, R. N. (1958b). Stimulus and response generalization: Tests of a model relating generalization to distance in psychological space. *Journal of Experimental Psychology*, *55*, 509–523.

Shepard, R. N. (1963). Analysis of proximities as a technique for the study of information processing in man. *Human Factors*, *5*, 33–48.

Shepard, R. N. (1964). Attention and the metric structure of the stimulus space. *Journal of Mathematical Psychology*, *1*, 54–87.

Shepard, R. N. (1965). Approximation to uniform gradients of generalization by monotone transformations of scale. In D. I. Mostofsky (Ed.), *Stimulus generalization* (pp. 94–110). Stanford, CA: Stanford University Press.

Shepard, R. N. (1966). Metric structure in ordinal data. *Journal of Mathematical Psychology*, *3*, 287–315.

Shepard, R. N. (1969, September). *Computer explorations of psychological space.* Paper presented at the Annual Meeting of the American Psychological Association.

Shepard, R. N. (1974). Representation of structure in similarity data: Problems and prospects. *Psychometrika*, *39*, 373–421.

Shepard, R. N., & Carroll, J. D. (1966). Parametric representation of nonlinear data structures. In P. R. Krishnaiah (Ed.), *Multivariate analysis*. New York: Academic.

Sherman, C. R. (1972). Nonmetric multidimensional scaling: A Monte-Carlo study of the basic parameters. *Psychometrika*, *37*, 323–355.

Shinew, K. J., Floyd, M., McGuire, F., & Noe, F. (1995). Gender, race, and subjective social class and their association with leisure preferences. *Leisure Studies*, *17*, 75–89.

Shye, S. (1981). Comparing structures of multidimensional scaling (SSA): A response to Kashti's study and to Langeheine's critique. *Studies in Educational Evaluation*, *7*, 105–109.

Shye, S. (1985). Nonmetric multivariate models for behavioral action systems. In D. Canter (Ed.), *Facet theory: Approaches to social research* (pp. 97–148). New York: Springer.

Shye, S. (1991). *Faceted SSA: A computer program for the PC* (Tech. Rep.). Jerusalem: The Louis Guttman Israel Institute of Applied Social Research.

Shye, S., Elizur, D., & Hoffman, M. (1994). *Facet theory.* Newbury Park, CA: Sage.

Sibson, R. (1979). Studies in the robustness of multidimensional scaling: Perturbation analysis of classical scaling. *Journal of the Royal Statistical Society*, *41*, 217–229.

Sixtl, F. (1967). *Messmethoden der Psychologie.* Weinheim, Germany: Beltz.

Sjöberg, L. (1975). Models of similarity and intensity. *Psychological Bulletin*, *82*, 191–206.

Snow, R. E., Kyllonen, P. C., & Marshalek, B. (1984). The topography of ability and learning correlations. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence* (Vol. 3, pp. 47–103). Hillsdale, NJ: Erlbaum.

Spence, I. (1972). *An aid to the estimation of dimensionality in nonmetric multidimensional scaling* (Tech. Rep. No. 229). University of Western Ontario Research Bulletin.

Spence, I. (1978). Multidimensional scaling. In P. W. Colgan (Ed.), *Quantitative ethology* (pp. 175–217). New York: Wiley.

Spence, I. (1979). A simple approximation for random rankings stress values. *Multivariate Behavioral Research*, *14*, 355–365.

Spence, I. (1983). Monte Carlo simulation studies. *Applied Psychological Measurement*, *7*(4), 405–425.

Spence, I., & Domoney, D. W. (1974). Single subject incomplete designs for nonmetric multidimensional scaling. *Psychometrika*, *39*, 469–490.

Spence, I., & Graef, J. (1974). The determination of the underlying dimensionality of an empirically obtained matrix of proximities. *Multivariate Behavioral Research*, *9*, 331–341.

Spence, I., & Ogilvie, J. C. (1973). A table of expected stress values for random rankings in nonmetric multidimensional scaling. *Multivariate Behavioral Research*, *8*, 511–517.

SPSS. (1990). *Categories.* Chicago: SPSS Inc.

Srinivasan, V., & Shocker, A. D. (1973). Linear programming techniques for multidimensional analysis of preferences. *Psychometrika*, *38*, 337–369.

Staufenbiel, T. (1987). Critical values and properties of μ_2. *Methodika*, *1*, 60–67.

Staufenbiel, T., & Borg, I. (1987). Dimensionen der Änlichkeit einfacher Reize: Breite-Höhe oder Grösse-Form? *Psychologische Beiträge*, *29*, 403–422.

Stenson, H. H., & Knoll, R. L. (1969). Goodness of fit for random rankings in Kruskal's nonmetric scaling procedure. *Psychological Bulletin*, *71*, 122–126.

Stephenson, W. (1953). *The study of behavior.* Chicago: University of Chicago Press.

Steverink, M., Van der Kloot, W., & Heiser, W. J. (2002). *Avoiding degenerate solutions in multidimensional unfolding by using additional distance information* (Tech. Rep. No. PRM 2002-01). Leiden University, the Netherlands: Department of Psychometrics.

Steyvers, M., & Busey, T. (2000). Predicting similarity ratings to faces using physical descriptions. In M. Wenger & J. Townsend (Eds.), *Computational, geometric, and process perspectives on facial cognition: Contexts and challenges* (pp. 115–146). New Jersey: Lawrence Erlbaum Associates.

Strang, G. (1976). *Linear algebra and its applications.* New York: Academic.

Suppes, P., & Zinnes, J. L. (1963). Basic measurement theory. In R. D. Luce, R. R. Bush, & E. Galanter (Eds.), *Handbook of mathematical psychology* (Vol. I, pp. 2–76). New York: Wiley.

Takane, Y., & Carroll, J. D. (1981). Nonmetric metric maximum likelihood multidimensional scaling from directional rankings of similarities. *Psychometrika*, *46*, 389–405.

Takane, Y., Young, F. W., & De Leeuw, J. (1977). Nonmetric individual differences multidimensional scaling: An alternating least-squares method with optimal scaling features. *Psychometrika*, *42*, 7–67.

Ten Berge, J. M. (1977). Orthogonal procrustes rotation for two or more matrices. *Psychometrika*, *42*, 267–276.

Ten Berge, J. M. F. (1991). A general solution for a class of weakly constrained linear regression problems. *Psychometrika*, *56*, 601–609.

Ten Berge, J. M. F. (1993). *Least squares optimization in multivariate analysis.* Leiden, The Netherlands: DSWO.

Ten Berge, J. M. F., & Kiers, H. A. L. (1991). Some clarifications of the CANDECOMP algorithm applied to INDSCAL. *Psychometrika*, *56*, 317–326.

Ten Berge, J. M. F., Kiers, H. A. L., & Commandeur, J. J. F. (1993). Orthogonal Procrustes rotation for matrices with missing values. *British Journal of Mathematical and Statistical Psychology*, *46*, 119–134.

Ten Berge, J. M. F., Kiers, H. A. L., & Krijnen, W. P. (1993). Computational solutions for the problem of negative saliences and nonsymmetry in INDSCAL. *Journal of Classification*, *10*, 115–124.

Tenenbaum, J. B. (1998). Mapping a manifold of perceptual observations. In M. Jordan, M. Kearns, & S. Solla (Eds.), *Advances in neural information processing 10* (pp. 682–687). Cambridge, MA.: MIT Press.

Tenenbaum, J. B., De Silva, V., & Langford, J. C. (2000). A global geometric framework for nonlinear dimension reduction. *Science*, *290*, 2319–2323.

Ter Braak, C. J. F. (1992). Multidimensional scaling and regression. *Statistica Applicata*, *4*, 577–586.

Thurstone, L. L. (1927). A law of comparative judgment. *Psychological Review*, *34*, 273–286.

Thurstone, L. L. (1935). *The vectors of mind.* Chicago: University of Chicago Press.

Thurstone, L. L. (1947). *Multiple factor analysis.* Chicago: University of Chicago Press.

Tobler, W., & Wineburg, S. (1971). A cappadocian speculation. *Nature*, *231*, 39–41.

Torgerson, W. S. (1952). Multidimensional scaling: I. Theory and method. *Psychometrika*, *17*, 401–419.

Torgerson, W. S. (1958). *Theory and methods of scaling.* New York: Wiley.

Torgerson, W. S. (1965). Multidimensional scaling of similarity. *Psychometrika*, *30*, 333–367.

Trosset, M. W. (1993). *A new formulation of the nonmetric Strain problem in multidimensional scaling* (Tech. Rep. No. TR93-30). Houston, TX: Department of Computational and Applied Mathematics, Rice University.

Tucker, L. R. (1951). *A method for the synthesis of factor analysis studies* (Tech. Rep. No. 984). Washington, DC: Department of the Army.

Tucker, L. R. (1960). Intra-individual and inter-individual multidimensionality. In H. Gulliksen & S. Messick (Eds.), *Psychological scaling: Theory and applications* (pp. 155–167). New York: Wiley.

Tucker, L. R. (1972). Relations between multidimensional scaling and three-mode factor analysis. *Psychometrika*, *37*, 3–27.

Tversky, A. (1977). Features of similarity. *Psychological Review*, *84*, 327–352.

Tversky, A., & Gati, I. (1982). Similarity, separability, and the triangle inequality. *Psychological Review*, *89*, 123–154.

Tversky, A., & Krantz, D. (1970). The dimensional representation and the metric structure of similarity data. *Journal of Mathematical Psychology*, *7*, 572–597.

Van der Heijden, P. G. M., De Falguerolles, A., & De Leeuw, J. (1989). A combined approach of contingency table analysis using correspondence analysis and loglinear analysis (with discussion). *Applied Statistics*, *38*, 249–292.

Van der Heijden, P. G. M., & De Leeuw, J. (1985). Correspondence analysis used complementary to loglinear analysis. *Psychometrika*, *50*, 429–447.

Van der Heijden, P. G. M., Mooijaart, A., & Takane, Y. (1994). Correspondence analysis and contingency table analysis. In M. J. Greenacre & J. Blasius (Eds.), *Correspondence analysis in the social sciences* (pp. 79–111). London: Academic.

Van der Lans, I. A. (1992). *Nonlinear multivariate analysis for multiattribute preference data.* Leiden, The Netherlands: DSWO.

Van Rijckevoorsel, J. L. A. (1987). *The application of fuzzy coding and horseshoes in multiple correspondence analysis.* Leiden, The Netherlands: DSWO.

Van Rijckevoorsel, J. L. A., & Tijssen, R. J. W. (1987). *Historical correspondence analysis: The history of correspondence analysis based on (co-)citations* (Tech. Rep. No. RR-87-11). Leiden, The Netherlands: Department of Data Theory, Leiden University.

Van Schuur, W. H. (1989). Unfolding the German political parties: A description and application of multiple unidimensional unfolding. In G. DeSoete, H. Feger, & K. C. Klauer (Eds.), *New developments in psychological choice modeling* (pp. 259–290). Amsterdam: Elsevier.

Van Schuur, W. H., & Post, W. J. (1990). *MUDFOLD manual.* Groningen, The Netherlands: IecProGAMMA.

Van Deun, K., Groenen, P. J. F., & Delbeke, L. (2005). *Vipscal: A combined vector ideal point model for preference data* (Tech. Rep. No. EI 2005-03). Econometric Institute Report.

Van Deun, K., Groenen, P. J. F., Heiser, W. J., Busing, F. M. T. A., & Delbeke, L. (2005). Interpreting degenerate solutions in unfolding by use of the vector model and the compensatory distance model. *Psychometrika*, *70*, 45–69.

Van Deun, K., Heiser, W. J., & Delbeke, L. (2004). Multidimensional unfolding by nonmetric multidimensional scaling of Spearman distances in the extended permutation polytope. (Unpublished manuscript)

Velleman, P., & Wilkinson, L. (1994). Nominal, ordinal, interval, and ratio typologies are misleading. In I. Borg & P. P. Mohler (Eds.), *Trends and perspectives in empirical social research* (pp. 161–177). New York: De Gruyter.

Verboon, P. (1994). *A robust approach to nonlinear multivariate analysis.* Leiden, The Netherlands: DSWO.

Verboon, P., & Heiser, W. J. (1992). Resistant orthogonal Procrustes analysis. *Journal of Classification*, *9*, 237–256.

Wagenaar, W. A., & Padmos, P. (1971). Quantitative interpretation of stress in Kruskal's multidimensional scaling technique. *British Journal of Mathematical and Statistical Psychology*, *24*, 101–110.

Wang, M. M., Schönemann, P. H., & Rusk, J. B. (1975). A conjugate gradient algorithm for the multidimensional analysis of preference data. *Multivariate Behavioral Research*, *10*, 45–79.

Weeks, D. G., & Bentler, P. M. (1979). A comparison of linear and monotone multidimensional scaling models. *Psychological Bulletin*, *86*, 349–354.

Wender, K. (1969). *Die psychologische Interpretation nichteuklidischer Metriken in der multidimensionalen Skalierung.* Unpublished doctoral dissertation, Technische Hochschule Darmstadt, Germany.

Wender, K. (1971). Die Metrik der multidimensionalen Skalierung als Funktion der Urteilsschwierigkeit. *Zeitschrift für experimentelle und angewandte Psychologie*, *18*, 166–187.

Wilkinson, L. (1990). *SYSTAT: The system for statistics.* Evanston, IL: SYSTAT Inc.

Wilkinson, L. (1996). Multidimensional scaling. In L. Wilkinson (Ed.), *Systat 6.0 for Windows: Statistics* (pp. 573–606). Chicago, IL: SPSS Inc.

Wilkinson, L., & Hill, M. (1994). *SYSTAT for DOS: Using SYSTAT, version 6 edition.* Evanston, IL: SYSTAT Inc.

Winsberg, S., & Carroll, J. D. (1989). A quasi- nonmetric method for multidimensional scaling via an extended Euclidean model. *Psychometrika*, *54*, 217–229.

Winsberg, S., & De Soete, G. (1993). A latent class approach to fitting the weighted Eulidean model, CLASCAL. *Psychometrika*, *58*, 315–330.

Winsberg, S., & De Soete, G. (1997). Multidimensional scaling with constrained dimensions. *British Journal of Mathematical and Statistical Psychology*, *50*, 55–72.

Wish, M. (1967). A model for the perception of Morse code like signals. *Human Factors*, *9*, 529–539.

Wish, M. (1971). Individual differences in perceptions and preferences among nations. In C. W. King & D. Tigert (Eds.), *Attitude research reaches new heights.* Chicago: American Marketing Association.

Wish, M., Deutsch, M., & Biener, L. (1970). Differences in conceptual structures of nations: An exploratory study. *Journal of Personality and Social Psychology*, *16*, 361–373.

Wish, M., Deutsch, M., & Biener, L. (1972). Differences in perceived similarity of nations. In A. K. Romney, R. N. Shepard, & S. B. Nerlove (Eds.), *Multidimensional scaling: Theory and applications in the behavioral sciences* (pp. 289–313). New York: Academic.

Wolford, G. L., & Hollingsworth, S. (1974). Evidence that short-term memory is not the limiting factor in the tachistoscopic full-report procedure. *Memory and Cognition*, *2*, 796–800.

Wolfrum, C. (1976a). Zum Auftreten quasiäquivalenter Lösungen bei einer Verallgemeinerung des Skalierungsverfahrens von Kruskal auf metrische Räume mit einer Minkowski-Metrik. *Archiv für Psychologie*, *128*, 96–111.

Wolfrum, C. (1976b). Zur Bestimmung eines optimalen Metrikkoeffizienten r mit dem Skalierungsverfahren von Kruskal. *Zeitschrift für experimentelle und angewandte Psychologie*, *23*, 339–350.

Woodworth, R. S. (1938). *Experimental psychology.* New York: Holt.

Xue, G. (1994). Improvement on the Northby algorithm for molecular confirmation: Better solutions. *Journal of Global Optimization*, *4*, 425–440.

Young, F. W. (1970). Nonmetric multidimensional scaling: Recovery of metric information. *Psychometrika*, *35*, 455–473.

Young, F. W. (1981). Quantitative analysis of qualitative data. *Psychometrika*, *46*, 357–388.

Young, F. W. (1984). The general Euclidean model. In H. G. Law, C. W. Snyder, Jr., J. A. Hattie, & R. P. McDonald (Eds.), *Research methods for multimode data analysis* (pp. 440–469). New York: Prager.

Young, F. W. (1987). Weighted distance models. In F. W. Young & R. M. Hamer (Eds.), *Multidimensional scaling: History, theory, and applications* (pp. 117–158). Hillsdale, NJ: Lawrence Erlbaum.

Young, G., & Householder, A. S. (1938). Discussion of a set of point in terms of their mutual distances. *Psychometrika*, *3*, 19–22.

Zielman, B., & Heiser, W. J. (1993). Analysis of asymmetry by a slide-vector. *Psychometrika*, *58*, 101–114.

Zielman, B., & Heiser, W. J. (1996). Models for asymmetric proximities. *British Journal of Mathematical and Statistical Psychology*, *49*, 127–146.

Zinnes, J. L., & MacKay, D. B. (1983). Probabilistic multidimensional scaling: Complete and imcomplete data. *Psychometrika*, *48*, 27–48.

Author Index

Subject Index