Chapter 10: Multidimensional Scaling

Multidimensional scaling (MDS) is a series of techniques that helps the analyst to identify key dimensions underlying respondents' evaluations of objects. It is often used in Marketing to identify key dimensions underlying customer evaluations of products, services or companies.

Once the data is in hand, multidimensional scaling can help determine:

- what dimensions respondents use when evaluating objects
- how many dimensions they may use in a particular situation
- the relative importance of each dimension, and
- how the objects are related perceptually

The purpose of MDS is to transform consumer judgments of similarity or preference (eg. preference for stores or brands) into distances represented in multidimensional space. The resulting **perceptual maps** show the relative positioning of all objects.

Multidimensional scaling is based on the comparison of **objects**. Any object (product, service, image, etc.) can be thought of as having both perceived and objective dimensions. For example, a firm may see their new model of lawnmower as having two color options (red versus green) and a 24-inch blade. These are the **objective dimensions**. Customers may or may not see these attributes. Customers may also perceive the lawnmower as expensive-looking or fragile, and these are the **perceived dimensions**.

- The dimensions perceived by customers may not coincide with (or even include) the objective dimensions assumed by the researcher.

- The evaluations of the dimensions may not be independent and may not agree. For example, one soft drink may be judged sweeter than another because the first has a fruitier aroma, although both contain the same amount of sugar.

Scenario Example

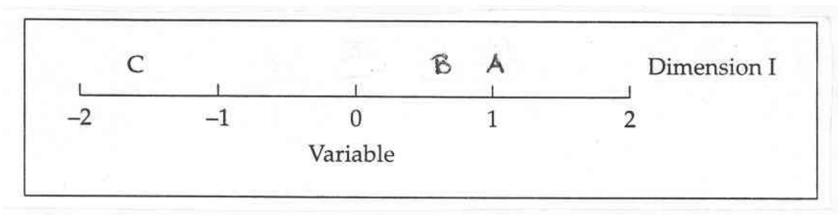
We are interested in understanding consumers' perceptions of six candy bars on the market. Instead of trying to gather information about consumers' evaluation of the candy bars on a number of attributes, the researcher will instead gather only perceptions of overall similarities or dissimilarities. The data are typically gathered by having respondents give simple global responses to statements such as these:

- Rate the similarity of products A and B on a 10-point scale
- Product A is more similar to B than to C
- I like product A better than product C

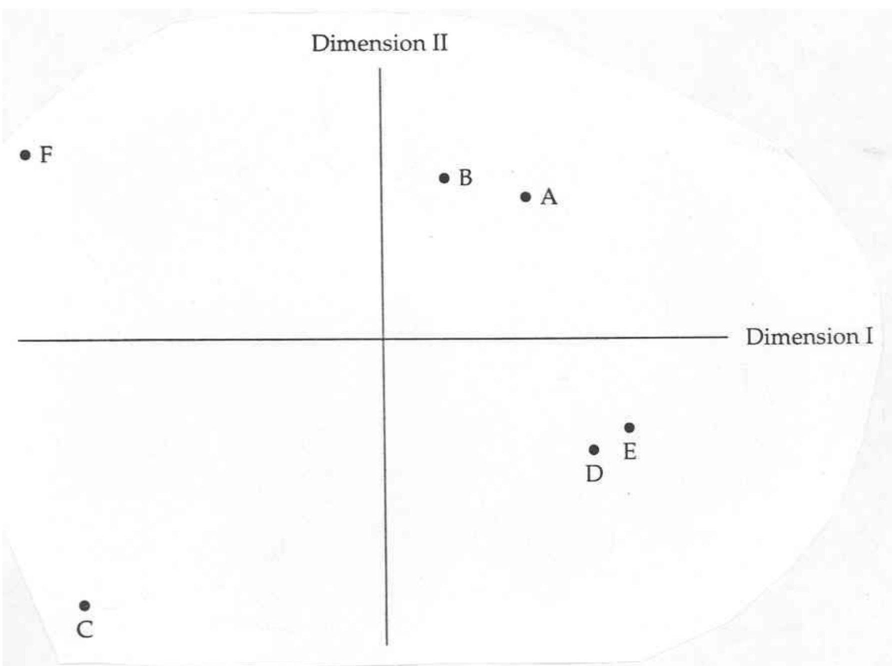
From these simple responses, a perceptual map can be drawn that best portrays the overall pattern of similarities among the six candy bars. The data are gathered by first creating a set of 15 unique pairs of the six candy bars (${}^6C_2$). Respondents are then asked to rank the following 15 candy bar pairs, where a rank of 1 is assigned to the pair of candy bars that is most similar and a rank of 15 indicates the pair is least alike. The results for all pairs of candy bars for one respondent are shown below:

Candy Bar	**A**	**B**	**C**	**D**	**E**	**F**
A	**_**	**2**	**13**	**4**	**3**	**8**
B		**_**	**12**	**6**	**5**	**7**
C			**_**	**9**	**10**	**11**
D				**_**	**1**	**14**
E					**_**	**15**
F						**_**

The respondent represented above thought that candy bars D and E were most similar, then A and B, with E and F the least similar. If we want to illustrate the similarity among candy bars graphically, a first attempt would be to draw a single **similarity scale**. We can do this for bars A, B and C as shown



Although a one-dimensional map can be accomplished with three objects, the task becomes increasingly difficult as the number of objects increases. Because one-dimensional scaling does not fit the data well, a two-dimensional solution should be attempted. This would allow for another scale (dimension) to be used in configuring the six candy bars, as shown:



The conjecture that at least two attributes (dimensions) were considered is based on the inability to represent the respondents perceptions in one dimension. However, we are still not aware of what attributes the respondent used in the evaluation.

Multidimensional scaling differs from the other interdependence techniques we have considered in two key aspects:

- Each respondent provides evaluations of all objects being considered, so that a solution can be obtained for each individual that is not possible in cluster analysis or factor analysis.
- Multidimensional scaling does not use a variate.

Step 1: Objectives Of Multidimensional Scaling

Perceptual mapping, and multidimensional scaling in particular, is most appropriate for achieving two objectives:

1. As an exploratory technique to identify unrecognized dimensions affecting behavior
2. As a means of obtaining comparative evaluations of objects when the specific bases of comparison are unknown or undefinable

The strength of perceptual mapping is its ability to infer dimensions without the need for defined attributes. In a simple analogy, it is like providing the dependent variable (similarity among objects) and figuring out what the independent variables (perceptual dimension) must be.

The researcher must define a multidimensional scaling analysis through three key decisions: selecting the objects that will be evaluated, deciding whether similarities or preference is to be analyzed and choosing whether the analysis will be performed at the group or individual level.

Identification Of All Relevant Objects To Be Evaluated

The most basic, but important, issue in perceptual mapping is the definition of the objects to be evaluated. The researcher must ensure that all "relevant" firms, products/services or other objects be included, and that no "irrelevant" objects are included, because perceptual mapping is a technique of relative positioning.

Similarity versus Preference Data

To this point we have discussed perceptual mapping and MDS mainly in terms of **similarity data**. In providing **preference data**, the respondent applies "good-bad" assessments, where we assume that differing combinations of perceived attributes are valued more highly than others. Both bases of comparison can be used to develop perceptual maps, but with differing interpretations.

Aggregate versus Disaggregate Analysis

In considering similarities or preference data, we are taking respondent's perceptions of different stimuli / treatments and creating outputs of the proximity of these treatments in t-dimensional space. The researcher can generate this output on a subject-by-subject basis (producing as many maps as subjects), known as **disaggregate analysis**. However, multidimensional scaling techniques can also combine respondents and create fewer perceptual maps by some process of **aggregate analysis**.

If the focus is on an understanding of the overall evaluations of objects and the dimensions employed in those evaluations, an aggregate analysis is the most appropriate. But if the objective is to understand variation among individuals, then a disaggregate approach is the most helpful.

Step 2: Research Design of MDS

Perceptual mapping techniques can be classified by the nature of the responses obtained from the individual concerning the object. One type, the **decompositional method**, measures only the overall impression or evaluation of an object and then attempts to derive spatial positions in multidimensional space reflecting these perceptions. The compositional method is an alternative method in which a defined set of attributes is considered in developing the similarity between objects.

Decompositional techniques are typically associated with multidimensional scaling and so our focus will be primarily on these methods.

Objects: Their Number and Selection

An implicit assumption in perceptual mapping is that there are common characteristics, either objective or perceived, that the respondent could use for evaluations. Therefore it is vital that the objects be comparable.

A second question deals with the number of objects to be evaluated. The researcher must balance two desires: a smaller number of objects to ease the effort on the part of the respondent versus the required number of objects to obtain a stable multidimensional solution. A suggested guideline for stable solutions is to have more than four times as many objects as dimensions desired.

Collection of Similarity or Preference Data

The primary distinction among multidimensional scaling programs is the type of data (qualitative or quantitative) used to represent similarity and preferences. For many of the data collection methods, either quantitative (ratings) or qualitative (rankings) data may be collected.

Similarities Data

When collecting similarities data, the researcher is trying to determine which items are the most similar to each other and which are the most dissimilar. Three procedures commonly used to obtain respondents' perceptions of similarities are outlined below:

- **Comparison Of Paired Objects:** By far the most widely used method of obtaining similarity judgments, the respondent is asked simply to rank or rate the similarity of all pairs of objects.

- **Confusion Data:** The pairing (or "confusing") of stimulus I with stimulus J is taken to indicate similarity. Also known as **subjective clustering**, the typical procedure for gathering these data is to place the objects whose similarity is to be measured (eg. ten candy bars) on small cards, either descriptively or with pictures. The respondent is asked to sort the cards into stacks so that all the cards in a stack represent similar candy bars. The data result in an aggregate similarities matrix similar to a cross-tabulation table.

- **Derived Measures:** These measures of similarity are typically based on scores given to stimuli by respondents. For example, subjects are asked to evaluate 3 stimuli (Pepsi, Coke and Allsport) on 2 semantic differential scales (Sweet to Tart, Light Tasting to Heavy). The 2×3 matrix could be evaluated for each respondent to create similarity measures.

There are several assumptions made with derived measures that make it the least desirable in meeting the “spirit” of MDS – that the evaluation of objects be made with minimal influence by the researcher.

Preference Data

Preference implies that stimuli should be judged in terms of dominance relationships – that is, stimuli are ordered in terms of the preference for some property. The two most common procedures for obtaining preference data are outlined below:

- **Direct Ranking:** Each respondent ranks the objects from most preferred to least preferred, as in the following example:

  Rank from most preferred (1) to least preferred (4)
  ______ Candy Bar A
  ______ Candy Bar B
  ______ Candy Bar C
  ______ Candy Bar D

- **Paired Comparisons:** A respondent is presented with all possible pairs and asked to indicate which member of each pair is preferred, as in this example:

  Please circle the preferred candy bar in each pair:
  A B
  A C
  A D
  B C
  B D
  C D

Preference data allows the researcher to view the location of objects in a perceptual map where difference implies differences in preference.

HATCO Example

The purpose of the research was to explore HATCO's image and competitiveness. This exploration included addressing the perceptions in the market of HATCO and nine major competitors, as well as an investigation of preferences among potential customers. The data were analyzed in a two-phase plan:

- Identification of the position of HATCO in a perceptual map of major competitors in the market
- Assessment of the preferences toward HATCO relative to major competitors

The HATCO image study comprised depth interviews with 18 midlevel management personnel from different firms selected as representative of the potential customer base existing in the market. The nine competitors, plus HATCO, represented all major firms in this industry and collectively had more than 85 percent of total sales. In the course of the interview, three types of data were collected: similarity judgments, attribute ratings of firms and preferences for each firm in different buying situations.

Similarity Data

The starting point for data collection was in obtaining the perceptions of the respondents concerning the similarity / dissimilarity of HATCO and nine competing firms in the market. Similarity judgments were made with the comparison-of-paired-objects approach. The 45 pairs of firms (${}^{10}C_2$) were presented to the respondents, who indicated how similar each was on a nine-point scale, with one being "Not at all similar" and nine being

“Very Similar.” Note that the values have to be transformed because increasing values for the similarity ratings indicate greater similarity, the opposite of a distance measure of similarity.

Attribute Ratings

In addition to the similarity judgments, ratings of each firm for eight attributes (product quality, delivery speed, etc.) were obtained by two methods. In the first method, each firm was rated on a six-point scale for each attribute. In the second method, each respondent was asked to pick the firm best characterized by each attribute. The respondent could pick any number of firms for each attribute. We will talk more about this later in the notes.

Preference Evaluations

The final data assessed the preferences of each respondent for the ten firms in three different buying situations: a straight re-buy, a modified re-buy and a new-buy situation. In each situation, the respondents ranked the firms in order of preference for that particular type of purchase.

Step 3: Assumptions of Multidimensional Scaling Analysis

Multidimensional scaling, while having no restraining assumptions on the methodology, type of data, or form of the relationships among the variables, does require that the researcher accept several tenets about perception, including the following:

1. Each respondent will not perceive a stimulus to have the same dimensionality (although it is thought that most people judge in terms of a limited number of characteristics or dimensions).

2. Respondents need not attach the same level of importance to a dimension, even if all respondents perceive this dimension.

3. Judgments of a stimulus in terms of either dimensions or levels of importance need not remain stable over time. People may not maintain the same perceptions for long periods of time.

HATCO Example (continued)

The assumptions of multidimensional scaling deal primarily with the comparability and representativeness of the objects being evaluated and the respondents. We obtained a representative sample of HATCO customers and care was taken to obtain respondents of comparable position and market knowledge.

Step 4: Deriving the MDS Solution and Assessing Overall Fit

The determination of how many dimensions are actually represented in the data is generally reached through one of three approaches: subjective evaluation, scree plots of the stress measures, or an overall index of fit.

One objective of the analyst should be to obtain the best fit with the smallest possible number of dimensions. Interpretation of solutions derived in more than three dimensions is extremely difficult and usually is not worth the improvement in fit. The analyst typically makes a **subjective evaluation** of the spatial maps and determines whether the configuration looks reasonable. This question must be considered, because at a later stage the dimensions will need to be interpreted and explained.

A second approach is to use a **stress measure**, which indicates the proportion of the variance of the **disparities** not accounted for by the MDS model. This measurement varies according to the type of program and the data being analyzed. Kruskal's stress is the most commonly used measure for determining a model's goodness of fit, and is provided in SPSS. Stress is minimized when the objects are placed in a configuration so that the distances between the objects best match the original distances.

A problem found in using stress, however, is analogous to that of R^2 in multiple regression in that stress always improves with increased dimensions. A trade-off must then be made between fit of the solution and the number of dimensions. We can plot the stress value against the number of factors to help us determine the optimal number of dimensions, in a similar technique to using a Scree Plot in Factor Analysis.

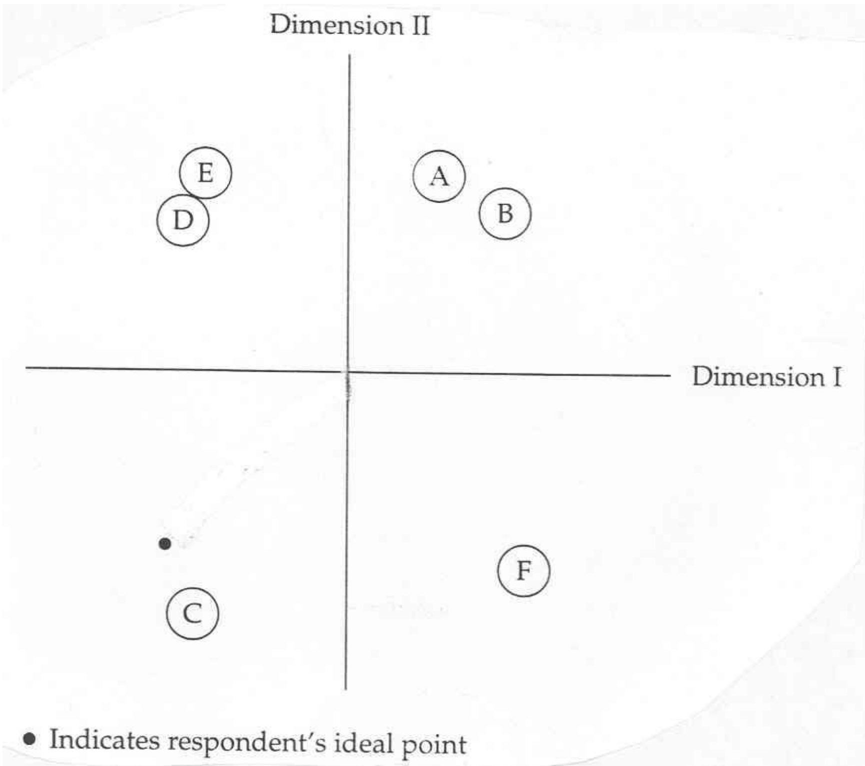
We can also use an R^2 measure as an **index of fit**, indicating the proportion of variance of the disparities accounted for by the MDS procedure.

Incorporating Preferences into MDS

Up to this point, we have concentrated on developing perceptual maps based on similarity judgments. However, perceptual maps can also be derived from preferences. A critical assumption is the homogeneity of perception across individuals for the set of objects. This allows all differences to be attributed to preferences, not perceptual differences.

We can assume that if we locate (on the derived perceptual map) the point that represents the most preferred combination of perceived attributes, we have identified the position of an ideal object. Equally we can assume that the position of this **ideal point** (relative to the other products on the derived perceptual map) defines relative preferences so that products farther from the ideal should be less preferred.

When preference data on the six candy bars (represented below) were obtained from a particular respondent, the point (.) was positioned so that increasing the distance from it indicated declining preference.



One may assume that this person's preference order is C, F, D, E, A, B. To imply that the ideal candy bar is exactly at the point (.) can be misleading. The ideal point simply defines the ordered preference relationship among the set of six candy bars for that respondent. Although ideal points individually may not offer much insight, clusters of them can be very useful in defining segments. Many respondents with ideal points in the same general area represent potential market segments of persons with similar preferences.

Explicit estimation can involve asking the subject to rate a hypothetical ideal on the same attributes on which the other stimuli are rated. Alternatively, the respondent is asked to include, among the stimuli used to gather similarities data, a hypothetical ideal stimulus (eg. brand, image). This is not ideal since respondents often conceptualize the ideal at the extremes of the explicit ratings used, or as being similar to the most preferred product from among those with which the respondent has had experience.

HATCO Example (continued)

Having specified the 10 firms to be included in the image study, HATCO's management specified that decompositional MDS be employed in constructing the perceptual maps [We will consider compositional Correspondence Analysis later].

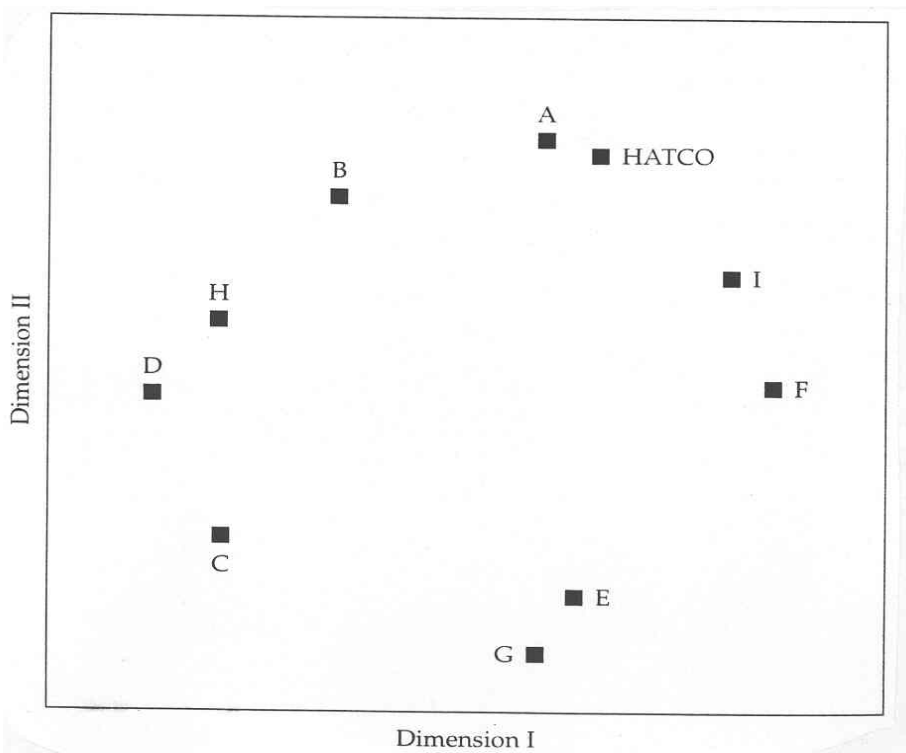
INDSCAL (Individual Differences Euclidean Distance Model) was used to develop the perceptual map. This type of analysis can be specified only if the analysis involves more than one data matrix (we have matrices for 18 respondents) and more than one dimension is desired. **EUCLID** (Euclidean Distance Model) could have been used if we were only concerned with individual matrices.

The first analysis of the MDS results is to determine the appropriate dimensionality and portray the results in a perceptual map. To do so, the researcher should consider the fit at each dimensionality and the researcher's ability to interpret the solution. The table below shows the fit for solutions of two to five dimensions.

Dimensions	**Stress**	**% Change**	**R^2**	**% Change**
5	.20068	_	.6303	_
4	.21363	6.4	.5557	11.8
3	.23655	10.7	.5007	9.9
2	.30043	27.0	.3932	21.5

There is a substantial improvement when moving from two to three dimensions, after which the improvement diminishes. Balancing this improvement in fit against the increasing difficulty of interpretation, the two- or three-dimensional solutions seem the most appropriate. For illustration purposes, we will now consider the two-dimensional solution.

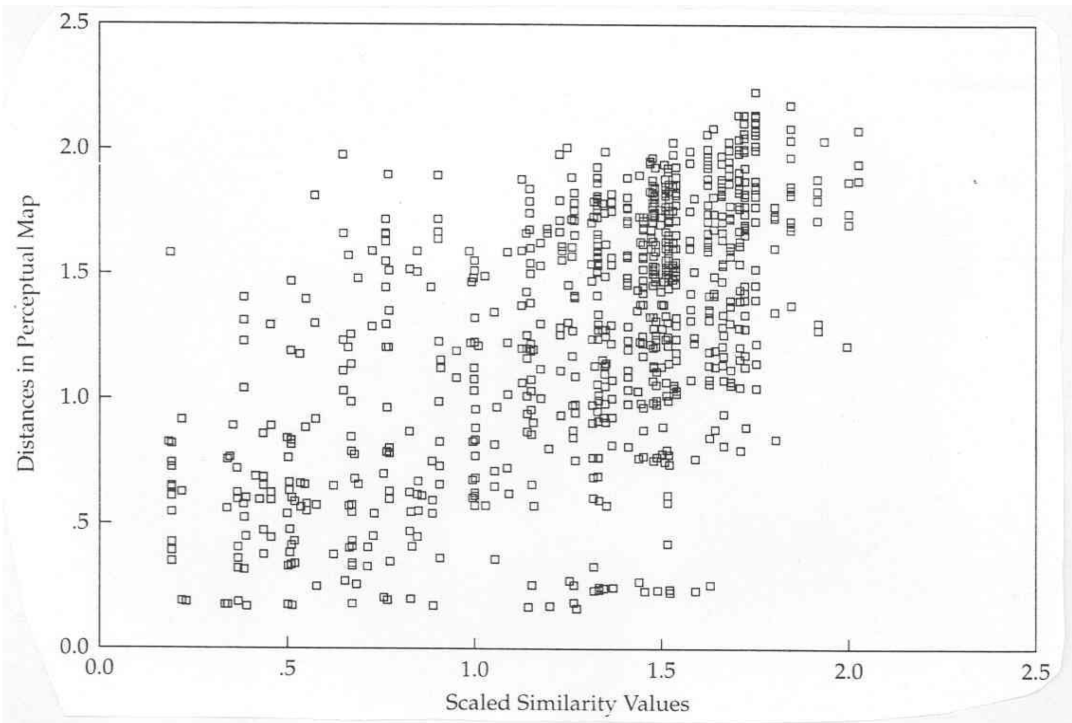
The two-dimensional aggregate perceptual map is shown below:



HATCO is most closely associated with firm A, with respondents considering them almost identical. Other pairs of firms considered highly similar based on their proximity are E and G, D and H, and F and I.

Comparisons can also be made between these firms and HATCO. HATCO differs from C, E and G primarily on dimension II, whereas dimension I differentiates HATCO most clearly from firms B, C, D and H in one direction and firms F and I in another direction. Similar comparisons can be made among all sets of firms. To understand the sources of these differences, however, the researcher must interpret the dimensions.

The researcher can also look at the fit of the solution in a scatterplot of actual distances (scaled similarity values) versus fitted distance from the perceptual map, shown below:



This plot can identify true outliers that are not well represented by the current solution. While the initial impression of this particular scatterplot is fairly poor, the density indicated by the plotting symbols reveals that the scatter is much more dense along the diagonal than away from it. Thus the scatter is not as poor as it originally appears.

If a consistent set of objects or individuals is identified as outliers, they can be considered for deletion. In this instance, no firm exhibits a large number of outlying points that would make it a candidate for elimination from the analysis.

In addition to developing the perceptual map, INDSCAL also provides the means for assessing the homogeneity of the respondents' perceptions. Weights are calculated for each respondent indicating the correspondence of their own perceptual space and the aggregate perceptual map. These weights provide a measure of comparison among respondents because respondents with similar weights have similar individual perceptual maps.

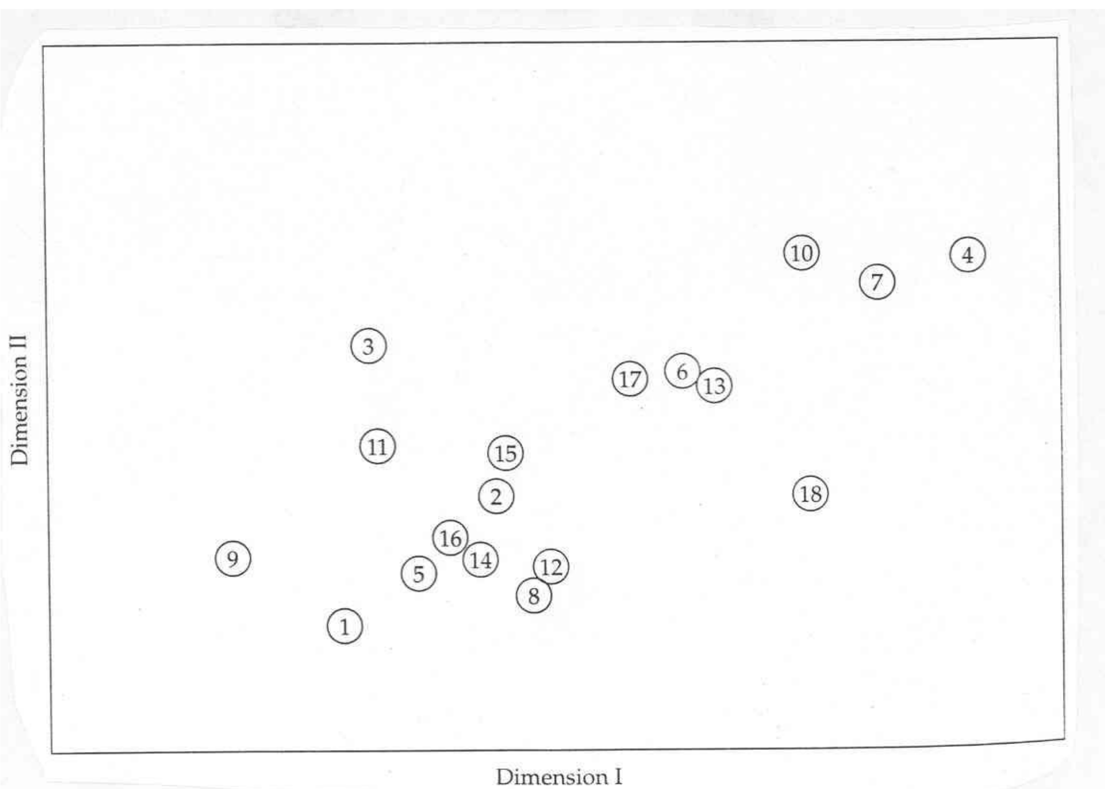
INDSCAL also produces a measure of fit for each subject by correlating the computed scores and the respondent's original similarity ratings. This output is given below:

		Dimension			
Subject Number	**Weird-ness**	**1**	**2**	**Stress**	**R^2**
1	.0163	.3864	.3534	.358	.274
2	.0034	.4322	.4077	.297	.353
3	.1527	.3946	.4717	.302	.378
4	.0322	.5724	.5106	.237	.588
5	.0138	.4089	.3755	.308	.308
6	.0052	.4876	.4612	.282	.450
7	.0169	.5458	.4988	.247	.547
8	.0801	.4438	.3671	.302	.332
9	.0899	.3537	.3824	.320	.271
10	.0249	.5235	.5108	.280	.535
11	.0902	.3966	.4290	.299	.341
12	.0678	.4476	.3776	.301	.343
13	.0142	.4969	.4560	.292	.455
14	.0325	.4273	.3810	.302	.328
15	.0263	.4356	.4260	.290	.371
16	.0037	.4183	.3902	.311	.327
17	.0204	.4724	.4578	.281	.433
18	.1187	.5253	.4086	.370	.443
Average				.300	.393

The **weirdness index** is designed to help interpret subject weights. The index indicates how unusual or weird each subject's weights are relative to the weights of the typical subject being analyzed. The index varies from 0 to 1.

A subject with a weirdness of 0 has weights that are proportional to the average subject's weights. As the weight ratios become more and more extreme, the weirdness index approaches 1. Finally, when a subject has only one positive weight and all the remaining weights are 0, the weirdness index is 1. Such a subject is very weird, using only one of the dimensions of the analysis.

Examination of the weights reveal that the respondents are quite homogeneous in their perceptions, because the weights show few substantive differences on either dimension and no distinct "clusters" of individuals emerge. Also, the maximum value from the weirdness index is only .1527.



Further confirmation is given in the plot of the weights above. All of the individual weights fall roughly on a straight line, indicating a consistent weight between dimensions I and II. The distance of each individual weight from the origin indicates its level of fit with the solution, with better fits shown by farther distances from the origin.

The fit values show relative consistency in both the stress and R^2 measures, with mean values of .300 (stress) and .393 (R^2). Moreover, all respondents are well represented, with the lowest level of fit being .27. Thus, no individual should be eliminated due to a poor fit in the two-dimensional solution.

Step 5: Interpreting The MDS Results

Once the perceptual map is obtained, the two approaches – compositional and decompositional – again diverge in their interpretation of the results. For compositional methods, the perceptual map must be validated against other measures of perception, because the positions are totally defined by the attributes specified by the researcher. For decompositional methods, the most important issue is the description of the perceptual dimensions and their correspondence to attributes.

Identifying The Dimensions

Multidimensional scaling techniques have no built-in procedure for labeling the dimensions. The researcher, having developed the maps with a selected dimensionality, can adopt several procedures, either subjective or objective.

Subjective Procedures

Interpretation must always include some element of researcher or respondent judgment, and in many cases this proves adequate for the questions at hand. A quite simple, yet effective, method is labeling (by visual inspection) the dimensions of the perceptual map by the respondent. Respondents may be asked to interpret the dimensionality subjectively by inspecting the maps, or a set of "experts" may evaluate and identify the dimensions. This approach may be the best available if the dimensions are believed to be highly intangible, or emotional in content, so that adequate descriptors cannot be devised.

In a similar manner, the researcher may describe the dimensions in terms of known (objective) characteristics.

Objective Procedures

As a compliment to the subjective procedures, a number of more formalized methods have been developed. The most widely used of these is PROFIT (**PRO**perty **FIT**ting), which collects attribute ratings for each object and then finds the best correspondence of each attribute to the derived perceptual space.

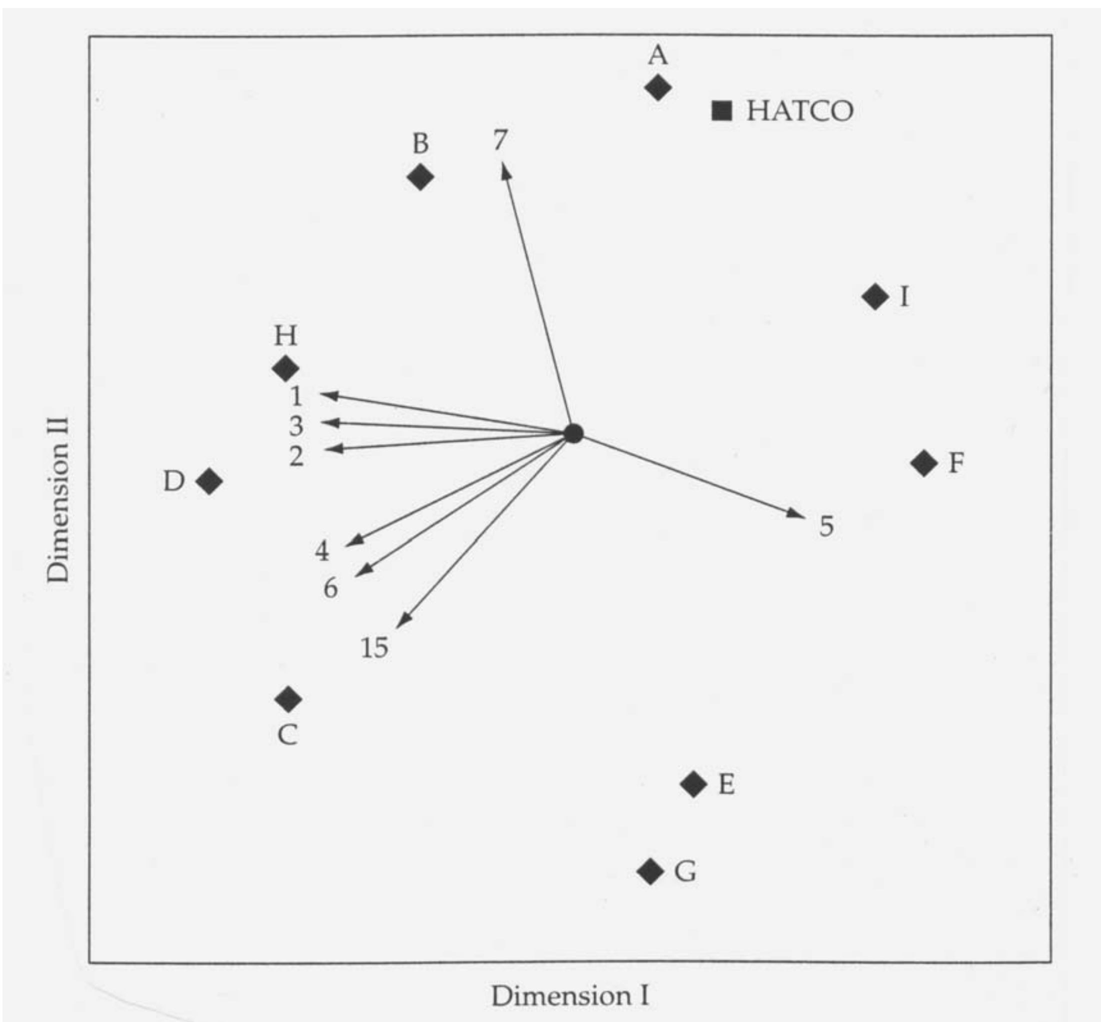
For either subjective or objective procedures, the researcher must remember that although a dimension can represent a single attribute, it usually does not. A more common procedure is to determine labels for each dimension using multiple attributes, similar to factor analysis. The problem, however, is that the researcher may not include all the important attributes in the study and so can never be totally assured that the labels represent all relevant attributes.

The task of labeling the axes cannot be left until completion, as the dimensional labels are essential for further interpretation and use of the results. Thus, the researcher must plan for the derivation of the dimensional labels as well as the estimation of the perceptual map.

Example (HATCO continued)

Once the perceptual map has been established, we can begin the process of interpretation. Because the INDSCAL procedure uses only the overall similarity judgments, HATCO also gathered ratings of the firms on eight attributes – the seven evaluations used before and a new variable, X_{15}, representing strategic orientation – descriptive of typical strategies followed in the industry.

PROFIT was used to match the ratings data to the firm positions in the perceptual map. The results of this are shown below:



We can see that there are three distinct "groups" or dimensions of attributes:

- The first involve X_1 (delivery speed), X_2 (price level) and X_3 (price flexibility), which are all pointed in the same direction, and X_5 (overall service), which is in the opposite direction. This directional difference indicates a negative correspondence of service versus the three other variables.

- The second set of variables reflects more global evaluations, consisting of the two image variables, X_4 and X_6, along with the new variable, X_{15} (strategic orientation).

- Finally, X_7 (product quality) runs almost perpendicular to the price and service dimension, indicating a separate and distinct evaluative dimension.

To interpret the dimensions, the researcher looks for attributes closely aligned with the axis. However, because the perceptual map is a point representation, the axes can we rotated without any impact on the relative positions. Rotating the axes slightly, we now have a dimension of price and service versus a second dimension of product quality.

Step 6: Validating the MDS Results

The most direct approach towards validation is a split-sample or multi-sample comparison, in which either the original sample is divided or a new sample is collected. Most often the comparison between results is done visually or with a simple correlation of coordinates.

Correspondence Analysis

Correspondence Analysis is an interdependence technique that has become increasingly popular for dimension reduction and perceptual mapping. It is a compositional technique because the perceptual map is based on the association between objects and a set of descriptive characteristics or attributes specified by the researcher. Its most direct application is portraying the "correspondence" of categories of variables, which is then used as the basis for developing perceptual maps.

Scenario Example

In its most basic form, Correspondence Analysis examines the relationships between categories of nominal data in a contingency table. For example, assume that sales figures for products A, B and C are broken down by three age categories. The data is shown below:

	Product Sales			
Age Category	**A**	**B**	**C**	**Total**
Young Adults (18-35 yrs old)	20	20	20	60
Middle Age (36-55 yrs old)	40	10	40	90
Senior Citizens (56+ yrs old)	20	10	40	70
Total	80	40	100	220

The data shows that unit sales vary substantially across products (product C has the highest total sales, product B the lowest) and age groups (middle age buys the most units, young adults the least). We may also want to examine whether the two factors are independent through the Chi Square Test.

Recall that the test statistic of the Chi-Square distribution is:

$$\chi^2 = \sum \frac{\left[n_{ij} - E(n_{ij})\right]^2}{E(n_{ij})}$$

where $E(n_{ij}) = \dfrac{\text{(Row Total)(Column Total)}}{\text{Total Sample Size}}$

It is possible to think of this statistic as the sum of a series of individual statistics. For example, the individual Chi-Square value for young adults buying product A would be:

$$\chi^2 = \frac{\left[n_{ij} - E(n_{ij})\right]^2}{E(n_{ij})} = \frac{\left[21.82 - 20\right]^2}{21.82} = .15$$

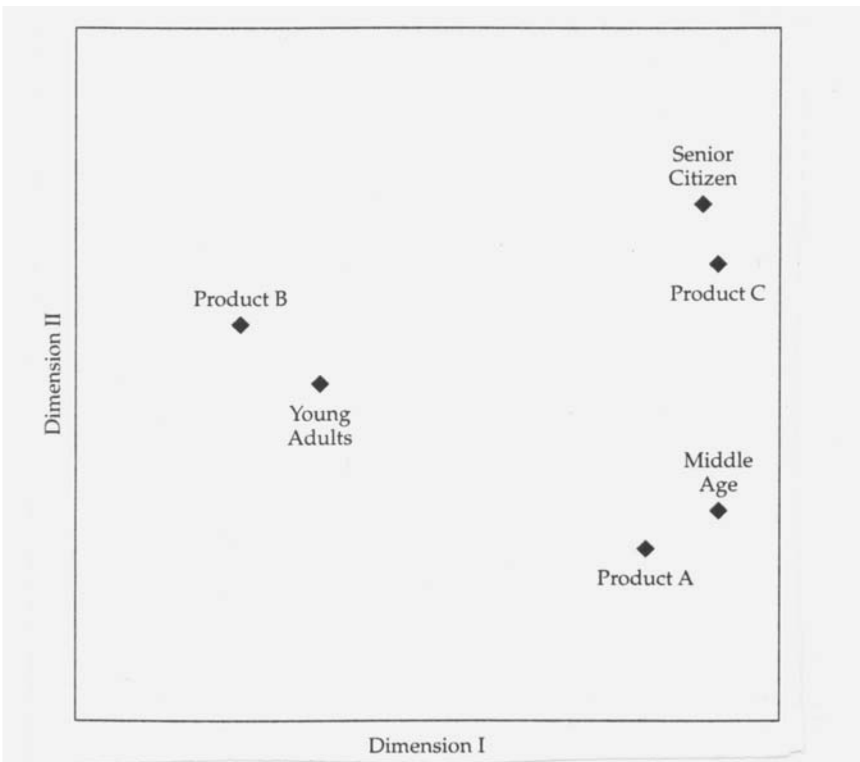
These individual Chi-Square values can be converted to similarity measures by applying the opposite sign of their difference. Thus the chi-square value above of .15 would be stated as a similarity value of -.15 because the difference was positive (ie. you expected more than you observed). These similarity values provide a standardized measure of association, much like the similarity judgments in the earlier candy bar example.

Our table of similarity values is as follows:

	Product Sales		
Age Category	**A**	**B**	**C**
Young Adults	-0.15	7.58	-1.94
Middle Age	1.62	-2.47	-0.02
Senior Citizens	-1.17	-0.58	2.10

With these association measures, correspondence analysis creates a quantitative distance measure and creates orthogonal dimensions upon which the categories can be placed to best account for the strength of association represented by the chi-square distances.

Note that the maximum number of dimensions is one less than the smaller of the number of rows or columns (so 3 – 1 = 2 in this example). The two-dimensional perceptual map is shown below.



From our analysis of the map we can see that young adults are closest to product B, the middle age group are closest to product A and the senior citizens are closest to product C. As with MDS, we do not know why the sales patterns existed, but only how to identify these patterns.

Step 1: Objectives Of Correspondence Analysis

Correspondence Analysis can address either of two basic objectives:

Association among row or column categories: CA can be used to examine the association among the categories of just a row or column. A typical use is in the examination of the categories of a scale. The categories can be compared to see if two can be combined (ie. they are in close proximity on the map) or if they do provide discrimination (ie. they are located separately in the perceptual space).

Association between row and column categories: In this application, interest lies in portraying the association between categories of the rows and columns, such as in our example of product by age group.

As a compositional method, the researcher must ensure that all the relevant variables appropriate for the research question have been included. This is in contrast to the decompositional MDS procedures described earlier, which require only the overall measure of similarity.

Step 2: Research Design of CA

Correspondence Analysis requires only a rectangular data matrix (contingency table) of non-negative entries. The categories for a row or column need not be a single variable but can represent any set of relationships. A prime example is the “pick any” method, in which respondents indicate which objects, if any, are described by the characteristics. Note that the respondent may choose any number of objects for each characteristic, rather than a pre-specified number.

Step 3: Assumptions in CA

Correspondence Analysis shares with the more traditional MDS techniques a relative freedom from assumptions. The use of strictly categorical data in a contingency table represents linear and nonlinear relationships equally well. The lack of assumptions, however, must not cause the researcher to neglect the efforts to ensure the comparability of objects and, because this is a compositional technique, the completeness of the attributes used.

Steps 4 & 5: Deriving CA Results, Assessing Overall Fit and Interpreting Results

As previously discussed we first must obtain a chi-square measure of similarity for the values in our contingency table. Once obtained, these chi-square values are standardized and converted to a distance value that will be represented in the perceptual map.

An SPSS program that will do this operation is ANACOR, although since we are using data in table form, this has to be performed using command syntax.

To assess overall fit, the researcher must first identify the appropriate number of dimensions and their importance. The maximum number of dimensions is one less than the smaller of the number of rows or columns, although as with MDS, a smaller number of dimensions aids interpretation.

SPSS also introduces a measure named "**inertia**," which also measures explained variation and is directly related to the eigenvalue. One rule of thumb is to include dimensions with eigenvalues greater than .2.

Once the dimensionality has been established, the researcher can identify a category's association with other categories by their proximity on the perceptual map.

Example (HATCO continued)

Preparing the data for analysis involves creating a cross-tabulation matrix relating the attributes (represented as rows) to the ratings of firms (the columns). The individual entries in the matrix are the number of times a firm is rated as possessing a specific attribute. This table is represented below:

	Firms									
Variables	*HATCO*	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
X_7 Product quality	4	3	1	13	9	6	3	18	2	10
X_{15} Strategic orientation	15	16	15	11	11	14	16	12	14	14
X_5 Overall service	15	14	6	4	4	15	14	13	7	13
X_1 Delivery speed	16	13	8	13	9	17	15	16	6	12
X_2 Price level	14	14	10	11	11	14	12	13	10	14
X_6 Salesforce image	7	18	13	4	9	16	14	5	4	16
X_3 Price flexibility	6	6	14	10	11	8	7	4	14	4
X_4 Manufacturer image	15	18	9	2	3	15	16	7	8	8

The ANACOR program in SPSS will take this table of frequencies and convert the values to Chi-Square similarity measures as discussed earlier. These values will then be standardized to give us the results in the table below:

TABLE 10.10 Measures of Similarity in CA: Chi-Square Distances

	Firms									
Variables	*HATCO*	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
X_7 Product quality	−1.27	−1.83	−2.08	3.19	1.53	−.86	−1.73	4.07	−1.42	.97
X_{15} Strategic orientation	.02	−.13	.76	−.01	.04	−.73	.07	−.60	1.07	−.20
X_5 Overall service	1.08	.40	−1.10	−1.52	−1.48	.57	.59	.65	−.36	.53
X_1 Delivery speed	.68	−.51	−.95	.95	−.27	.40	.20	.86	−1.15	−.37
X_2 Price level	.19	−.19	−.30	.37	.42	−.30	−.54	.08	.20	.23
X_6 Salesforce image	−1.32	−1.49	1.15	−1.54	.23	.81	.55	−1.80	−1.44	1.39
X_3 Price flexibility	−1.02	−1.28	2.37	1.27	1.71	−.73	−.83	−1.59	2.99	−1.66
X_4 Manufacturer image	1.24	1.69	−.01	−2.14	−1.76	.72	1.32	−1.07	.10	−.85

High positive values indicate a strong degree of "correspondence" between the attribute and firm, with negative values having an opposite interaction. CA tries to satisfy all of these relationships simultaneously by producing dimensions representing the chi-square distances.

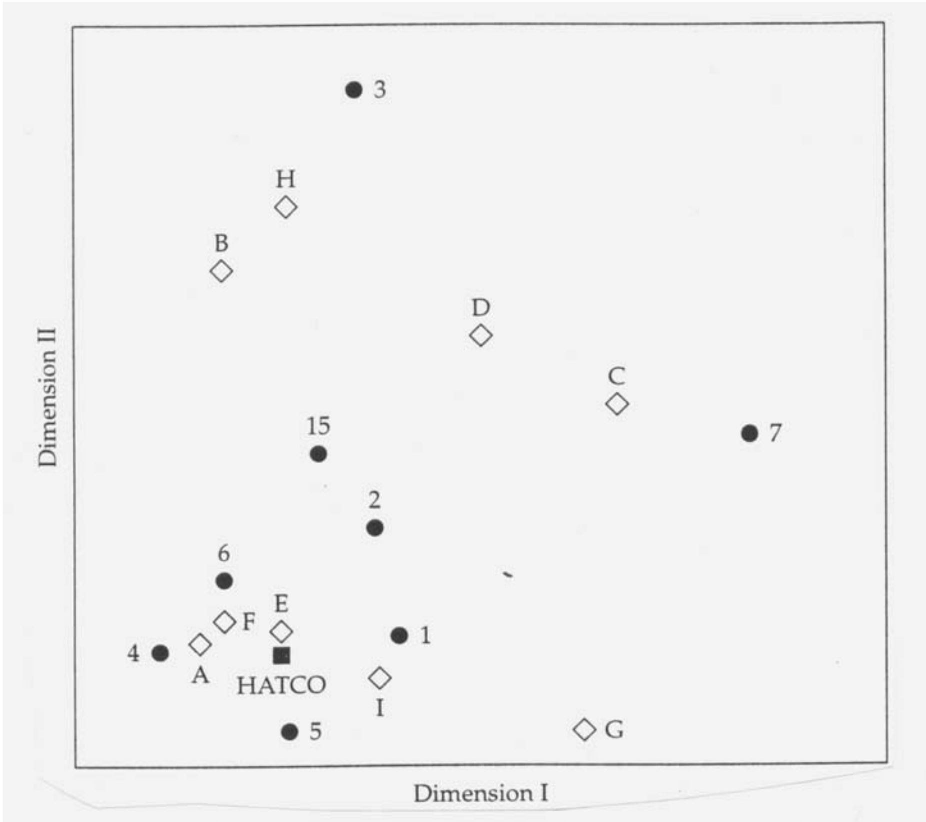
The table below contains the eigenvalues and explained variation for each dimension:

TABLE 10.11 Determining the Appropriate Dimensionality in CA

Dimension	*Eigenvalue (Singular Value)*	*Inertia (Normalized Chi-Square)*	*Percentage Explained*	*Cumulative Percentage*
1	.27666	.07654	53.1	53.1
2	.21866	.04781	33.2	86.3
3	.12366	.01529	10.6	96.9
4	.05155	.00266	1.8	98.8
5	.02838	.00081	.6	99.3
6	.02400	.00058	.4	99.7
7	.01951	.00038	.3	100.0

A two-dimensional solution in this situation explains 86% of the variation, whereas increasing to a three-dimensional solution adds only an additional 10%. Due to reasons of interpretability, a two-dimensional solution is deemed appropriate for further analysis.

The attribute-based perceptual map below shows the relative proximities of both firms and attributes:



Focusing on the firms first, we see that the pattern of firm groups is similar to that found in MDS. Firms A, E, F and I, plus HATCO form one group; firms C and D and firms H and B form two other similar groups.

	Coordinates		**Contribution to Inertia**	
	I	**II**	**I**	**II**
X_1	.204	-.245	.022	.040
X_2	.115	.046	.007	.001
X_3	.044	1.235	.001	.689
X_4	-.676	-.285	.196	.044
X_5	-.202	-.502	.018	.142
X_6	-.440	-.099	.087	.006
X_7	1.506	.298	.665	.033
X_{15}	-.081	.245	.004	.045

Explanation by Dimension

	I	II	Total
X_1	.289	.330	.619
X_2	.469	.058	.527
X_3	.002	.989	.991
X_4	.789	.111	.901
X_5	.138	.677	.816
X_6	.358	.014	.372
X_7	.961	.030	.991
X_{15}	.093	.678	.772

For the attributes, we can see that X_7 (product quality) is the primary contributor to dimension I, with X_4 a secondary contributor. Between these two attributes, 86% of dimension I is accounted for. A similar pattern follows for dimension II, for which X_3 (price flexibility) is the primary contributor, followed by X_5 (overall service).

Explanation of the dimensions by the firms can be examined in a similar way. [Output for firms is included in the textbook]

Step 6: Validation of the Results

As with all MDS techniques, an emphasis must be made to ensure generalizability through split-sample or multi-sample analyses. However, as with other perceptual mapping techniques, the generalizability of the objects (individually and as a set) must also be established.

The sensitivity of the results to the addition or deletion of an object can be evaluated, as well as the addition or deletion of an attribute. The goal is to assess whether the analysis is dependent on only a few objects and/or attributes. In either instance, the researcher must understand the "true" meaning of the results in terms of the objects and attributes.

Example (HATCO continued)

Perhaps the strongest validation of this analysis is to assess the convergence between the results from the separate decompositional and compositional techniques.

When our perceptual maps from MDS and CA are compared, they show quite similar patterns of firms reflecting two groups: firms B, H, D and C versus firms E, F, G and I. While the relative distances among firms do vary between the two perceptual maps, we still see HATCO associated strongly with firms A and I in each perceptual map. CA produces more distinction between the firms, but its objective is to define firm positions as a result of differences; thus it will generate more distinctiveness in its perceptual maps. The interpretation of axes also shows similar patterns.