Truman Lee Kelley (1884–1961)

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This talk and the associated handout are at:

`cda.psych.uiuc.edu/kelley_handout.pdf`

`cda.psych.uiuc.edu/kelley_beamer_talk_psychometric_society.pdf`

Kelley's Educational Background at Illinois

Kelley was one of the most prominent psychometricians (or, for that matter, statisticians) from the first half of the twentieth century.

We give some facts gleaned from various obituaries of Kelley, and also from his transcript at the University of Illinois (they gave me a copy because of my faculty status at Illinois).

From his transcript we know he was born in Whitehall, Michigan (May 25, 1884).

His father, M.C. Kelley, was a lawyer, who lived at 135 Muskegon Avenue in Muskegon, Michigan.

We don't know who his mother was; there is a blank line on the Illinois transcript that says: Name of Mother, if Father is dead.

Kelley started at Illinois in 1904; he received an A.B. degree in 1909 with Special Honors in Mathematics.

B. S. Thesis title: *Graphic evaluation of trigonometric functions of complex variables*

He received an A.M. degree in Psychology in 1911; he is listed as an Assistant helping teach Experimental Psychology (Psychology 3 and 4) in the 1910–11 Catalog.

M. A. Thesis title: *Correlation between certain mental capacities*

This appeared in Psychological Review in 1913 under the title *The association experiment: Individual differences and correlations*

A footnote to this article reads: This experiment was conducted at the University of Illinois in 1911 and the complete data, including a record of the 1,200 introspections, is to be found on file in the library of the university.

The only explicit course I could see on Kelley's transcript that related to statistics was Math 14, Method of Least Squares, taken in 1906.

We note that in the 1905–6 Catalog, an applied class in Statistics was offered (called Statistical Adjustments) that was directed towards students in Economics and Zoology. It was taught by Assistant Professor Rietz.

In the 1910–11 Catalog, Mathematics 129, Theory of Statistics, was listed but only offered in 1911–12, again by Assistant Professor Rietz; presumably this was after Kelley had left.

I believe it was Rietz that had the most influence on Kelley's statistical interests; also, Karl Pearson appears to have been a major influence (Kelley spent a sabbatical year with Pearson in 1922–23 at the Galton Biometric Laboratory).

We quote from the Preface of Kelley's *Statistical Method* (1923) (the Charles C. Grove mentioned was a mathematician at Columbia where Kelley did his doctoral degree under Thorndike):

I would, however, say that my greatest inspiration has been the product of that master analyst, Karl Pearson, and that the English school entire has been most contributive. My greatest indebtedness to men in America is to my teachers, Henry Lewis Rietz and Charles C. Grove, for enlightenment upon theoretical points and to Edward L. Thorndike for suggestions as to problems in need of statistical analysis.

Julian Stanley in a biographical piece in *JEBS* referred to Kelley as "Stanford University's Pearson" ...

After Illinois

Kelley received his Ph.D. in 1914 with E.L. Thorndike at Columbia.

His thesis was entitled *Educational Guidance*, and was an extremely heavy computational study for the time, with multiple regressions and multiple correlations galore.

This would characterize Kelley's work throughout his career:

1) a very heavy multivariate computational burden;
2) the development of innovative iterative algorithms;
3) significant (educational) psychology applications;
4) various forms and versions of the Kelley Statistical Tables.

What is a Computer?

Truman Lee Kelley (1884–1961)

Lawrence Hubert

Remember what a "computer" meant in the early 1900s –

We quote from the preface of *Educational Guidance*:

To the ever-ready stimulating criticisms of Professor Thorndike, I am peculiarly indebted, for it is due to his encouragement that the investigation covers the three fields of mathematics, English, and history instead of one only, and that the number of relations determined is as extensive as it is. The field covered gives the work whatever of value it has, but the accomplishment of it and its appearance in print at this time has been possible only because of the devoted and untiring assistance, in grading, calculating coefficients of correlation, and deriving regression equations, rendered by my wife.

September, 1914

Past the Ph.D.

After his Ph.D. Kelley wandered around a bit:

University of Texas (1914–17);

Columbia University (1917–20);

during this time he was a psychological consultant to the US Army Committee on Classification of Personnel and to the Surgeon General's Office.

Here, he used the statistical treatment of data to predict, by means of psychological tests, the performance of men on one job or another.

Kelley moved to Stanford in 1920 as a Professor of Education and Psychology.

With Lewis Terman he first published the Stanford Achievement Test Battery (1922), versions of which are still in use today (10th edition now).

Kelley moved to Harvard's School of Education in 1931 and stayed till he retired in 1950.

Books by Kelley

Seven Kelley books we will cite:

Statistical Method (1923)

Interpretation of Educational Measurements (1927)

Crossroads in the Mind of Man: A Study of Differential Mental Abilities (1928)

Essential Traits of Mental Life (1935)

Talents and Tasks: Their Conjunction in a Democracy for Wholesome Living and National Defense (1940)

Fundamentals of Statistics (1947)

The Kelley Statistical Tables (1948)

Kelley Honors

Psychometric Society President (1938–9; after Thurstone, Thorndike, and Guilford, but before Holzinger)

Vice-President of the American Statistical Association (1926)

Chair of Section Q (Education) of the AAAS (1928)

Co-founder (in 1904 with William Bagley at Illinois) of the national honor society in education – Kappa Delta Pi

Topics in the Handout

Truman Lee Kelley (1884–1961)

Lawrence Hubert

1) Principal component approaches:

Hotelling's power method

Kelley's rotations to the major and minor axes of an ellipse

2) Kelley True Score Predictions (James-Stein Estimation)

A telling quote is given from Stephen Stigler's 1988 Neyman Memorial Lecture, "A Galtonian Perspective on Shrinkage Estimators" (1990, *Statistical Science*)

3) Kelley's iterative rotation strategy for canonical correlation analysis

4) Two items about Kelley's Will from the Associated Press.

This Will set up eugenic fitness tests for his sons and future daughter-in-laws

5) Parts of an interview with Darrell Bock in JEBS (2006)

6) Parts of an interview with Gene Golub for SIAM (2006)

A quote from "Estimation and Tests of Significance in Factor Analysis," C. Radhakrishna Rao, Visiting Research Professor, Department of Psychology, University of Illinois (*Psychometrika*, 1955):

Even with a good set of trial values the problem can be best tackled only on an electronic computer when large numbers of variables are involved. A suitable program for Illiac is being written by Mr. Golub of the Digital Computer Laboratory at the University of Illinois. A numerical example solved on a tentative program is reported below. Full details will be presented soon.

Kelley Contributions

Things for which Kelley was either there first or at the same time as someone else:

1) James-Stein (shrunken) estimators (this is from his 1923 book, *Statistical Method*)

2) Principal components (this is from his 1935 text, *Essential Traits of Mental Life*)

3) Canonical correlation and canonical variables (this is from his 1940 book, *Talents and Tasks*)

4) Unbiased estimation of the Correlation Ratio ("An unbiased correlation ratio measure." *Proceedings of the National Academy of Sciences*, 1935)

5) Asymptotic variance formulas for tetrad differences using the delta method (this is from his 1928 text, *Crossroads in the Mind of Man*; also, see the later slides on J.L. Doob)

6) Pentad conditions for factor analysis (also from *Crossroads in the Mind of Man*)
(Spearman): Four variables may be thought of as due to one general factor plus four specific factors when

$$r_{12}r_{34} = r_{13}r_{24} = r_{14}r_{23}$$

Or, when we have three tetrad differences being equal to zero.
(Kelley) Five variables may be thought of as due to two general factors plus five specific factors when a pentad criterion is zero. For terms such as $r_{12}r_{13}r_{24}r_{35}r_{45}$, six are added together and six subtracted.

7) From Campbell and Fiske's classic 1959 *Psychological Bulletin* article on multitrait-multimethod matrices:

Multitrait-multimethod matrices are rare in the test and measurement literature ... One of the earliest matrices of this kind was provided by Kelley and Krey in 1934. Peer judgments by students provided one method, scores on a word-association test the other.

8) Contributions to Classical Test Theory (quotations from Traub's article, Classical Test Theory in Historical Perspective (1997)):

The index of reliability and other results. It is a worthwhile experience, though in the late 20th century a humbling one, to read the articles on test theory that were published between 1910 and 1925 by Spearman, Brown, and Kelley, among others. These documents contain a great many of the basic results of classical test theory. Kelley's 1923 test, Statistical Methods, included a compilation of these results in a section on reliability theory. Kelley also stated in this text the definition of reliability that he championed throughout his long career: the coefficient of correlation between "comparable tests."

An important result, used by Spearman in his 1910 proof of the prophecy formula, was the expression for the correlation between two composite measures in terms of the variances and covariances of the components. From this result, Abelson (1911) derived the formula for what came to be known several years later as the index of reliability. It seems this name was coined quite by accident. Kelley independently derived the formula for the index in 1916 and then in using it wrote that "the extent to which the grade determined by means of this test of forty words would correlate with the true spelling ability of the individual is probably an even more significant index of reliability."

Rietz Handbook

H.L Rietz (Editor), *Handbook of Mathematical Statistics* (1924); Chapters by:

Members of the Committee on the Mathematical Analysis of Statistics of the Division of Physical Sciences of the National Research Council. This included A.R. Crathorne, H.C. Carver, and Truman Lee Kelley (Chapter IX: Partial and Multiple Correlation).

Rietz was a reviewer of *Statistical Method* (1923). We give a few interesting quotes:

In harmony with this view the book gives a large number of determinations of probable errors. The determinations of these probable errors was surely a very difficult undertaking on the part of the author and he should be complimented on his courage.

...

Special attention should thus be directed to the methods which facilitate the computations of partial and multiple correlation coefficients, including the method of successive approximation devised by Dr. Kelley for finding the regression coefficients when a large number of variables are involved.

An aside on Joseph Leo Doob (1910–2004) (aka Mr. Martingale)

Ph.D. Harvard (1932)

Illinois Mathematics Department (1935–1978)

Two of his students:

Paul Halmos (known for the text *Finite Dimensional Vector Spaces* and the "halmos," the little tombstone at the end of a proof)

David Blackwell (the first African-American inducted into the National Academy of Sciences, and the first black tenured faculty member at UC Berkeley)

From the Wikipedia article on Doob:

The Great Depression of 1929 was still going strong in the thirties and Doob could not find a job. B. O. Koopman at Columbia University suggested that statistician Harold Hotelling might have a grant that would permit Doob to work with him. Hotelling did, so the Depression led Doob to probability.

One of Doob's early publications was "The Limiting Distributions of Certain Statistics" (*The Annals of Mathematical Statistics*, 1935), listing him at Columbia and with the acknowledgement "Research under a grant-in-aid from the Carnegie Corporation." We give the abstract and a few interesting excepts from this piece:

Truman Lee Kelley (1884–1961)

Lawrence Hubert

There have been many advances in the theory of probability in recent years, especially relating to its mathematical basis. Unfortunately, there appears to be no source readily available to the ordinary American statistician which sketches these results and shows their application to statistics. It is the purpose of this paper to define the basic concepts and state the basic theorems of probability, and then, as an application, to find the limiting distributions for large samples of a large class of statistics. One of these statistics is the tetrad difference, which has been of much concern to psychologists.

Truman Lee Kelley (1884–1961)

Lawrence Hubert

There is a well-known δ-method used in statistics to find limiting variances of statistics of the type covered by Theorem 1, and Theorem 1 shows an interpretation which can be given to the results obtained by this method. ... Examples of the use of this method can be found in T. L. Kelley, Crossroads in the Mind of Man, Stanford University (1928), pp. 49-50, and in an article by S. Wright, Annals of Mathematical Statistics, Vol. 5 (1934), p. 211.

Two quotes from Bock, Multivariate Statistical Methods in Behavioral Research (1975)

Truman Lee Kelley (1884–1961)

Lawrence Hubert

The Jacobi method. Another iterative method for finding the characteristic roots and vectors of real symmetric matrices is implicit in a procedure, proposed by Jacobi in 1846, for improving the numerical conditioning of least-squares equations. Jacobi's procedure was all but forgotten until rediscovered by Kelley (1935) as a method of principal factor analysis and revived independently by Goldstine, Murray, and von Neumann (1969) as an algorithm for machine computation. (p. 97)

Truman Lee Kelley (1884–1961)

Lawrence Hubert

The Most Predictable Criterion: Canonical Correlation — Responding to a query from Truman Kelley, Harold Hotelling in 1936 investigated the problem of finding the linear combination of criteria that has the greatest multiple correlation with the predictors. The solution to this problem that Hotelling published in the 1935 volume of the *Journal of Educational Psychology* was the basis for his formulation of canonical correlation as a general method for analyzing linear relations between two sets of variables (Hotelling, 1936; *Biometrika*) (p. 389)

A Quote From Kelley

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Lawrence Hubert

The opening sentences from Kelley's, *Essential Traits of Mental Life* (The Purposes and Principles Underlying the Selection and Measurement of Independent Mental Factors, Together with Computational Tables):

A New Method of Analysis of Variables into Independent Components: Before attempting a comparison of different methods of analysis of variables into components, a new method is presented. The procedure followed is new, but the outcome is identical with that given by Hotelling's method of analysis.

The Crux of Kelley's Approach to Principal Components

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Lawrence Hubert

From the Handout:
... that one [] considered to have special merit [] is a rotation of the x and y axes to the position of the major and minor axes of the ellipse. These particular new variables, which we designate x_1 and y_1, are given by the equations

$$x_1 = x \cos\theta + y \sin\theta$$

$$y_1 = -x \sin\theta + y \cos\theta$$

where θ is the angle of rotation and is given by

$$\tan 2\theta = \frac{2p}{v_1 - v_2}$$

$[p = \sigma_{12};\ v_1 = \sigma_1^2;\ v_2 = \sigma_2^2]$

Kelley did not "rediscover" Jacobi's method (he did not know, for example, that multiplying the pairwise orthogonal rotations together would give the eigenvectors directly as is done in Jacobi's method).

Instead, Kelly got it from Pearson's 1901 paper, "On lines and planes of closest fit in systems of points in space," where the "tangent formula" appears directly.

About the same time that Kelley published a general solution for the principal axes problem in *Essential Traits of Mental Life* (1935), Harold Hotelling (with whom Kelley consulted) also published such a solution using what is typically called the power method.

Hotelling's 1933 paper(s) in the *Journal of Educational Psychology*, "Analysis of a Complex of Statistical Variables into Principal Components," begins with this footnote:

A study made in part under the auspices of the Unitary Traits Committee and the Carnegie Corporation.

The author is indebted to Professor Truman L. Kelley, who was responsible for the initiation of this study and the propounding of many of the questions to which answers are here attempted; also to Professors L. L Thurstone, Clark V. Hull, C. Spearman, and E. L. Thorndike, who raised some of the further questions treated.

The group to which Hotelling refers was part of what was called the Unitary Traits Committee (chaired by Thorndike; formed in 1931; funded by the American Council of Education)

Kelley and Hotelling constituted the subcommittee on "mathematical theory and techniques and the improvement of methods of analysis."

In the very first issue of *Psychometrika* in 1936, Harold Hotelling has an article entitled "Simplified Calculation of Principal Components." We quote from the introduction:

Another method of calculating principal components has been discovered by Professor Truman L. Kelley, which involves less labor than the original iterative method, at least in the examples to which he has applied it ... How it would compare with the present accelerated method is not clear, except that some experience at Columbia University has suggested that the method here set forth is the more efficient. It is possible that Kelley's method is more suitable when all the characteristic roots are desired, but not the corresponding correlations of the variates with the components.

An aside on Gene Golub (1932–2007)

Truman Lee Kelley (1884–1961)

Lawrence Hubert

One of the preeminent numerical analysts of his generation.

Ph.D. from Illinois in 1959 under Abraham Taub (in Statistics).

On his advisor: "I was subject to a lot of abuse by Taub. He would just yell and scream at me. He was really a nasty piece."

He had better luck with the psychologists at Illinois — Charles Wrigley, in particular.

He also worked for C.R. Rao when Rao was a visitor in the Psychology Department; he programmed Rao's Maximum Likelihood Factor Analysis for the Illiac.

In Darrell Bock's interview from the handout, Bock mentions coming down to Illinois (from the University of Chicago) in the 1950's and meeting Golub.

Golub had programmed the eigenroutines for the Illiac and helped Bock with his analyses on a Discriminant Analysis problem.

Bock recognized the Jacobi methods used for the Illiac as what Kelley had done — "I now believe that he rediscovered Jacobi's method independently."

Well, not quite. Kelley took the principal axis rotational solution (for two variables) from Pearson's 1901 paper; he did not know that multiplying the transformations together would then converge to the eigenvectors.

Although Hotelling is viewed as the author of Principal Components analysis, his method of computation involving squaring a matrix to speed up convergence is not used today. In fact, analogues of Kelley's rotation strategy are more popular (e.g., the Jacobi method).

We quote from Bodewig's *Matrix Calculus*:

Powers of Matrices: Many authors such as Kincaid, Aitken, Hammersley, and Hotelling, recommend successive squaring of $\mathbf{A}$ and iteration with $\mathbf{A}^{2m}$ on $\mathbf{v}$ instead of with $\mathbf{A}$ itself. This is done in order to speed up convergence and to save work. But this proposal cannot be defended.