Truman Lee Kelley (1884–1961)

Lawrence Hubert

Department of Psychology
The University of Illinois

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This talk and the associated handout are at:

`cda.psych.uiuc.edu/kelley_handout.pdf`

`cda.psych.uiuc.edu/kelley_beamer_talk.pdf`

Kelley's Educational Background at Illinois

Kelley was one of the most prominent psychometricians (or, for that matter, statisticians) from the first half of the twentieth century.

We give some facts gleaned from various obituaries of Kelley and from his transcript at the University of Illinois.

From his transcript we know he was born in Whitehall, Michigan (May 25, 1884).

His father, M.C. Kelley, was a lawyer, who lived at 135 Muskegon Avenue in Muskegon, Michigan.

We don't know who his mother was; there is a blank line on the Illinois transcript that says: Name of Mother, if Father is dead.

Kelley started at Illinois in 1904; he received an A.B. degree in 1909 with Special Honors in Mathematics.

He received an A.M. degree in Psychology in 1911; he is listed as an Assistant helping teach Experimental Psychology (Psychology 3 and 4) in the 1910–11 Catalog.

The only explicit course I could see on Kelley's transcript that related to statistics was Math 14, Method of Least Squares, taken in 1906.

We note that in the 1905–6 Catalog, an applied class in Statistics was offered (called Statistical Adjustments) that was directed towards students in Economics and Zoology. It was taught by Assistant Professor Rietz.

In the 1910–11 Catalog, Mathematics 129, Theory of Statistics, was listed but only offered in 1911–12, again by Assistant Professor Rietz; presumably this was after Kelley had left.

I believe it was Rietz that had the most influence on Kelley's statistical interests; also, Karl Pearson appears to have been a major influence (Kelley spent a sabbatical year with Pearson in 1922–23 at the Galton Biometric Laboratory).

We will have more to say about Rietz later.

We quote from the Preface of Kelley's *Statistical Method* (1923) (the Charles C. Grove mentioned was a mathematician at Columbia where Kelley did his doctoral degree under Thorndike):

I would, however, say that my greatest inspiration has been the product of that master analyst, Karl Pearson, and that the English school entire has been most contributive. My greatest indebtedness to men in America is to my teachers, Henry Lewis Rietz and Charles C. Grove, for enlightenment upon theoretical points and to Edward L. Thorndike for suggestions as to problems in need of statistical analysis.

After Illinois

Kelley received his Ph.D. in 1914 with E.L. Thorndike at Columbia.

His thesis was entitled *Educational Guidance*, and was an extremely heavy computational study for the time, with multiple regressions and multiple correlations galore.

This would characterize Kelley's work throughout his career:

1) a very heavy multivariate computational burden;
2) the development of innovative iterative algorithms;
3) significant (educational) psychology applications;
4) various forms and versions of the Kelley Statistical Tables.

What is a Computer?

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Remember what a "computer" meant in the early 1900s –

We quote from the preface of *Educational Guidance*:

To the ever-ready stimulating criticisms of Professor Thorndike, I am peculiarly indebted, for it is due to his encouragement that the investigation covers the three fields of mathematics, English, and history instead of one only, and that the number of relations determined is as extensive as it is. The field covered gives the work whatever of value it has, but the accomplishment of it and its appearance in print at this time has been possible only because of the devoted and untiring assistance, in grading, calculating coefficients of correlation, and deriving regression equations, rendered by my wife.

September, 1914

Kelley's' Grave in Santa Barbara

Truman Lee Kelley
1884 — 1961

Grace Winifred Kelley
1898 — 1989

Given that Kelley finished his thesis in 1914, can we conclude that this wife, Grace Kelley, was not his computer?

Past the Ph.D.

After his Ph.D. Kelley wandered around a bit:

University of Texas (1914–17);

Columbia University (1917–20);

during this time he was a psychological consultant to the US Army Committee on Classification of Personnel and to the Surgeon General's Office.

Here, he used the statistical treatment of data to predict, by means of psychological tests, the performance of men on one job or another.

Kelley moved to Stanford in 1920 as a Professor of Education and Psychology.

With Lewis Terman he first published the Stanford Achievement Test Battery (1922), versions of which are still in use today (10th edition now).

Kelley moved to Harvard's School of Education in 1931 and stayed till he retired in 1950.

Books by Kelley

Seven Kelley books we will cite:

Statistical Method (1923)

Interpretation of Educational Measurements (1927)

Crossroads in the Mind of Man: A Study of Differential Mental Abilities (1928)

Essential Traits of Mental Life (1935)

Talents and Tasks: Their Conjunction in a Democracy for Wholesome Living and National Defense (1940)

Fundamentals of Statistics (1947)

The Kelley Statistical Tables (1948)

Kelley Honors

Psychometric Society President (1938–9; after Thurstone, Thorndike, and Guilford, but before Holzinger)

Vice-President of the American Statistical Association (1926)

Chair of Section Q (Education) of the AAAS (1928)

Co-founder (in 1904 with William Bagley at Illinois) of the national honor society in education – Kappa Delta Pi

Topics in the Handout

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1) Principal component approaches:

Hotelling's power method

Kelley's rotations to the major and minor axes of an ellipse

2) Kelley True Score Predictions (James-Stein Estimation)

A telling quote is given from Stephen Stigler's 1988 Neyman Memorial Lecture, "A Galtonian Perspective on Shrinkage Estimators" (1990, *Statistical Science*)

3) Kelley's iterative rotation strategy for canonical correlation analysis

4) Two items about Kelley's Will from the Associated Press.

This Will set up eugenic fitness tests for his sons and future daughter-in-laws

5) Parts of an interview with Darrell Bock in JEBS (2006)

6) Parts of an interview with Gene Golub for SIAM (2006)

A quote from "Estimation and Tests of Significance in Factor Analysis," C. Radhakrishna Rao, Visiting Research Professor, Department of Psychology, University of Illinois (*Psychometrika*, 1955):

Even with a good set of trial values the problem can be best tackled only on an electronic computer when large numbers of variables are involved. A suitable program for Illiac is being written by Mr. Golub of the Digital Computer Laboratory at the University of Illinois. A numerical example solved on a tentative program is reported below. Full details will be presented soon.

Kelley Contributions

Things for which Kelley was either there first or at the same time as someone else:

1) James-Stein (shrunken) estimators (this is from his 1923 book, *Statistical Method*)

2) Principal components (this is from his 1935 text, *Essential Traits of Mental Life*)

3) Canonical correlation and canonical variables (this is from his 1940 book, *Talents and Tasks*)

4) Unbiased estimation of the Correlation Ratio ("An unbiased correlation ratio measure." *Proceedings of the National Academy of Sciences*, 1935)

5) Asymptotic variance formulas for tetrad differences using the delta method (this is from his 1928 text, *Crossroads in the Mind of Man*; also, see the later slides on J.L. Doob)

6) Pentad conditions for factor analysis (also from *Crossroads in the Mind of Man*)
(Spearman): Four variables may be thought of as due to one general factor plus four specific factors when

$$r_{12}r_{34} = r_{13}r_{24} = r_{14}r_{23}$$

Or, when we have three tetrad differences being equal to zero.
(Kelley) Five variables may be thought of as due to two general factors plus five specific factors when a pentad criterion is zero. For terms such as $r_{12}r_{13}r_{24}r_{35}r_{45}$, six are added together and six subtracted.

Henry Lewis Rietz (1875–1943)

Cornell Ph.D. in Mathematics in 1902;

He came to the Illinois math department in 1903 and stayed till 1918.

In 1905 he was appointed as a statistician in the College of Agriculture – split his time between mathematics and that college until he left Illinois.

Rietz's first publication in the statistical field under his name was a 32-page appendix to Dean (of Agriculture) Davenport's treatise on breeding (1907).

In 1916 he became the actuary for the Illinois Pension Laws Commission, and was very active in actuarial things thereafter.

We quote from a Memorial in the *Annals* (1944) written by A.R. Crathorne (1873–1946) (one of Hilbert's students – 1907, who came to Illinois in 1908), another Illinois faculty member in math when Rietz was here (and a long time collaborator with Rietz on many textbooks):

In 1918 he was called to the University of Iowa as head of the department of mathematics ...

Rietz was head of mathematics at Iowa till 1943; Allen Craig (1931) and Samuel Wilks (1931) were his students, among others.

Rietz was a founder of the Institute of Mathematical Statistics (and of the *Annals of Mathematical Statistics*).

Rietz Handbook

H.L Rietz (Editor), *Handbook of Mathematical Statistics* (1924); Chapters by:

Members of the Committee on the Mathematical Analysis of Statistics of the Division of Physical Sciences of the National Research Council. This included A.R. Crathorne, H.C. Carver, and Truman Lee Kelley (Chapter IX: Partial and Multiple Correlation).

Rietz was a reviewer of *Statistical Method* (1923). We give a few interesting quotes:

In harmony with this view the book gives a large number of determinations of probable errors. The determinations of these probable errors was surely a very difficult undertaking on the part of the author and he should be complimented on his courage.

A Misguided Interpretation

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From a history item about Kappa Delta Pi and Truman Lee Kelley:

Kelley felt that the basic statistical needs of the two fields were the same. Because he wanted to provide a foundation for advanced work for scholars, he asked fellow statisticians to look for probable errors in his work.

Joseph Leo Doob (1910–2004) (aka Mr. Martingale)

Ph.D. Harvard (1932)

Illinois Mathematics Department (1935–1978)

Two of his students:

Paul Halmos (known for the text *Finite Dimensional Vector Spaces* and the "halmos," the little tombstone at the end of a proof)

David Blackwell (the first African-American inducted into the National Academy of Sciences, and the first black tenured faculty member at UC Berkeley)

From the Wikipedia article on Doob:

The Great Depression of 1929 was still going strong in the thirties and Doob could not find a job. B. O. Koopman at Columbia University suggested that statistician Harold Hotelling might have a grant that would permit Doob to work with him. Hotelling did, so the Depression led Doob to probability.

One of Doob's early publications was "The Limiting Distributions of Certain Statistics" (*The Annals of Mathematical Statistics*, 1935), listing him at Columbia and with the acknowledgement "Research under a grant-in-aid from the Carnegie Corporation." We give the abstract and a few interesting excepts from this piece:

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There have been many advances in the theory of probability in recent years, especially relating to its mathematical basis. Unfortunately, there appears to be no source readily available to the ordinary American statistician which sketches these results and shows their application to statistics. It is the purpose of this paper to define the basic concepts and state the basic theorems of probability, and then, as an application, to find the limiting distributions for large samples of a large class of statistics. One of these statistics is the tetrad difference, which has been of much concern to psychologists.

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There is a well-known δ-method used in statistics to find limiting variances of statistics of the type covered by Theorem 1, and Theorem 1 shows an interpretation which can be given to the results obtained by this method. ... Examples of the use of this method can be found in T. L. Kelley, Crossroads in the Mind of Man, Stanford University (1928), pp. 49-50, and in an article by S. Wright, Annals of Mathematical Statistics, Vol. 5 (1934), p. 211.

Two quotes from Bock, Multivariate Statistical Methods in Behavioral Research (1975)

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The Jacobi method. Another iterative method for finding the characteristic roots and vectors of real symmetric matrices is implicit in a procedure, proposed by Jacobi in 1846, for improving the numerical conditioning of least-squares equations. Jacobi's procedure was all but forgotten until rediscovered by Kelley (1935) as a method of principal factor analysis and revived independently by Goldstine, Murray, and von Neumann (1969) as an algorithm for machine computation. (p. 97)

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The Most Predictable Criterion: Canonical Correlation — Responding to a query from Truman Kelley, Harold Hotelling in 1936 investigated the problem of finding the linear combination of criteria that has the greatest multiple correlation with the predictors. The solution to this problem that Hotelling published in the 1935 volume of the *Journal of Educational Psychology* was the basis for his formulation of canonical correlation as a general method for analyzing linear relations between two sets of variables (Hotelling, 1936; *Biometrika*) (p. 389)

A Quote From Kelley

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The opening sentences from Kelley's, *Essential Traits of Mental Life* (The Purposes and Principles Underlying the Selection and Measurement of Independent Mental Factors, Together with Computational Tables):

A New Method of Analysis of Variables into Independent Components: Before attempting a comparison of different methods of analysis of variables into components, a new method is presented. The procedure followed is new, but the outcome is identical with that given by Hotelling's method of analysis.

The Crux of Kelley's Approach to Principal Components

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From the Handout:
... that one [] considered to have special merit [] is a rotation of the x and y axes to the position of the major and minor axes of the ellipse. These particular new variables, which we designate x_1 and y_1, are given by the equations

$$x_1 = x \cos\theta + y \sin\theta$$

$$y_1 = -x \sin\theta + y \cos\theta$$

where θ is the angle of rotation and is given by

$$\tan 2\theta = \frac{2p}{v_1 - v_2}$$

$[p = \sigma_{12};\ v_1 = \sigma_1^2;\ v_2 = \sigma_2^2]$

Kelley did not "rediscover" Jacobi's method (he did not know, for example, that multiplying the pairwise orthogonal rotations together would give the eigenvectors directly as is done in Jacobi's method).

Instead, Kelly got it from Pearson's 1901 paper, "On lines and planes of closest fit in systems of points in space," where the "tangent formula" appears directly.

From J.C. Flanagan's Harvard thesis under Kelley (*Factor Analysis in the Study of Personality*, 1935)

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This brings us directly to the last method of multiple-factor analysis which we shall consider, that of Hotelling. At the request of the Unitary Traits Committee, Hotelling attacked the problem of obtaining a serviceable solution to the problem proposed by Kelley in 1928, "first, a determination, having tests A, B, C, of what the independent mental traits are; and secondly an experimental construction of new tests measuring these independent traits." As we have just noted, Hotelling's least-squares conditions are identical to those in one of the solutions presented by Thurstone. Dr. Hotelling, however, has supplied a very neat iterative solution of the kth order determinant involved which makes the solution comparatively short.

Harold Hotelling (1895–1973)

Born in Fulda, Minnesota

Stanford (1927–31) [Kelley was at Stanford from 1920 to 1931; in his 1927 book *Interpretation of Educational Measurements*, Kelley notes in the preface: "I am indebted to my colleague(s), Dr. Harold Hotelling, for a suggestion followed in Section 5 of Chapter VIII" [this was a suggestion on how to evaluate a particular determinant]

Columbia (1931–46) [E.L. Thorndike was there at the time]

North Carolina (1946–73)

About the same time that Kelley published a general solution for the principal axes problem in *Essential Traits of Mental Life* (1935), Harold Hotelling (with whom Kelley consulted) also published such a solution using what is typically called the power method.

Hotelling's 1933 paper(s) in the *Journal of Educational Psychology*, "Analysis of a Complex of Statistical Variables into Principal Components," begins with this footnote:

A study made in part under the auspices of the Unitary Traits Committee and the Carnegie Corporation.

The author is indebted to Professor Truman L. Kelley, who was responsible for the initiation of this study and the propounding of many of the questions to which answers are here attempted; also to Professors L. L Thurstone, Clark V. Hull, C. Spearman, and E. L. Thorndike, who raised some of the further questions treated.

The group to which Hotelling refers was part of what was called the Unitary Traits Committee (chaired by Thorndike; formed in 1931; funded by the American Council of Education)

Kelley and Hotelling constituted the subcommittee on "mathematical theory and techniques and the improvement of methods of analysis."

We quote from Karl Holzinger's "Recent Research on Unitary Mental Traits" (*Journal of Personality*, 1935):

THE COMMITTEE ON MATHEMATICAL METHODS

During the early meetings of the Unitary Traits Committee some criticism was made of existing methods of factorization, chiefly those of Professor Kelley in *Crossroads in the Mind of Man*. Professor Kelley was already at work amending these techniques, and enlisted the aid of Professor Harold Hotelling to further this work. As a mathematical statistician Professor Hotelling was of great service to the committee. He contributed many valuable suggestions at meetings, and the factorization technique now known as the Method of Principle [sic] Components.

Very recently Professor Kelley has published a volume entitled *The Essential Traits of Mental Life* (1935). In this book he has contributed a method of factorization which appears simpler than that of Hotelling, but which gives the same results. In addition to this new technique Professor Kelley makes a comparison of current methods of factorization. He also presents the view that "a trait becomes a trait because of a social recognition of it, and the more it is recognized and valued by society, the more of a trait it is." Starting with this point of view, he presents an analysis of occupational groups from the United States Census classifications of 1930.

In the very first issue of *Psychometrika* in 1936, Harold Hotelling has an article entitled "Simplified Calculation of Principal Components." We quote from the introduction:

Another method of calculating principal components has been discovered by Professor Truman L. Kelley, which involves less labor than the original iterative method, at least in the examples to which he has applied it ... How it would compare with the present accelerated method is not clear, except that some experience at Columbia University has suggested that the method here set forth is the more efficient. It is possible that Kelley's method is more suitable when all the characteristic roots are desired, but not the corresponding correlations of the variates with the components.

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The present method seems to the computers who have tried both to be superior when the components themselves, as well as their contributions to the total variance, are to be specified. The advantage of the present method is enhanced when, as will often be the case in dealing with numerous variates, not all the characteristic roots but only a few of the largest are required.

Iterative processes of various kinds are capable of acceleration by means of the matrix-squaring device here used.

Gene Golub (1932–2007)

One of the preeminent numerical analysts of his generation.

Ph.D. from Illinois in 1959 under Abraham Taub (in Statistics).

On his advisor: "I was subject to a lot of abuse by Taub. He would just yell and scream at me. He was really a nasty piece."

He had better luck with the psychologists at Illinois — Charles Wrigley, in particular.

He also worked for C.R. Rao when Rao was a visitor in the Psychology Department; he programmed Rao's Maximum Likelihood Factor Analysis for the Illiac.

In Darrell Bock's interview from the handout, Bock mentions coming down to Illinois (from the University of Chicago) in the 1950's and meeting Golub.

Golub had programmed the eigenroutines for the Illiac and helped Bock with his analyses on a Discriminant Analysis problem.

Bock recognized the Jacobi methods used for the Illiac as what Kelley had done — "I now believe that he rediscovered Jacobi's method independently."

Well, not quite. Kelley took the principal axis rotational solution (for two variables) from Pearson's 1901 paper; he did not know that multiplying the transformations together would then converge to the eigenvectors.

Some Trivia

My copy of Kelley's book *Statistical Method* (1923) has the signature of Dael Wolfle, with the date of 1928. This was when Dael was 22.

Dael lead AAAS and *Science* in the 1950s and 1960s.

Under Thurstone, he wrote the *Psychometrika* (1940) monograph, "Factor Analysis to 1940." He refers to the "Hotelling and Kelley method" to reference principal components.

Harold Rugg

In my copy of Kelley's *The Influence of Nurture Upon Native Differences* (1926), we have the handwritten note:

To
Dr. Harold O. Rugg with the compliments of the author
T.L.K.

From the Wikipedia entry on Rugg:

Harold Rugg (1886-1960) was an educational reformer in the early to mid 1900s, associated with the Progressive Education Movement. Originally trained in civil engineering at Dartmouth College (BS 1908 & CE 1909), Rugg went on to study psychology, sociology and education at the University of Illinois where he completed a dissertation titled "The Experimental Determination of Mental Discipline in School Studies." After earning his Ph.D. he went on to teach at the University of Chicago and later became a professor at Teacher's College at Columbia University.

He studied the creativity which he believed was vital to the learning process. He created the first textbook series and his social studies books were extremely popular in US schools. By the early forties his books fell out of favor due to a campaign run by big business organizations like the Advertising Federation of America (AFA) and even the American Legion. The cause for the controversy was that in some of Rugg's junior-high textbooks he had included some pro-socialist ideas and included some criticism of the AFA.

Although Hotelling is viewed as the author of Principal Components analysis, his method of computation involving squaring a matrix to speed up convergence is not used today. In fact, analogues of Kelley's rotation strategy are more popular (e.g., the Jacobi method).

We quote from Bodewig's *Matrix Calculus*:

Powers of Matrices: Many authors such as Kincaid, Aitken, Hammersley, and Hotelling, recommend successive squaring of $\mathbf{A}$ and iteration with $\mathbf{A}^{2m}$ on $\mathbf{v}$ instead of with $\mathbf{A}$ itself. This is done in order to speed up convergence and to save work. But this proposal cannot be defended.

My Favorable Experiences with Gene Golub

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These are based on two papers:

M.T. Chu, R.E. Funderlic, and G.H. Golub. A rank-one reduction formula and it applications to matrix factorizations. SIAM Review, 1995.

Lawrence Hubert, Jacqueline Meulman, and Willem Heiser. Two Purposes for Matrix Factorization: A Historical Appraisal. SIAM Review, 2000.

This later paper points out the historical fact that Louis Guttman did in 1944 (*Psychometrika*) what Chu et al. did in 1995.